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Numerical Computation of Steady Buoyancy Driven MHD Heat and Mass Transfer Past An Inclined Infinite Flat Plate with Sinusoidal Surface Boundary Conditions

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Abstract

In this paper we study the effects of magnetohydrodynamics (MHD) fluid flow on a two dimensional boundary layer flow of a steady free convection heat and mass transfer on an inclined plate in which the angle of inclination is varied. The fluid is taken as viscous, incompressible, electrically conducting. The mathematical formulation yields a set of governing partial differential equations ($PDEs$) under a set of appropriate boundary conditions. The $PDEs$ are transformed into ordinary differential equations ($ODEs$) by some similarity transformation. The $ODEs$ are solved using the shooting method with the fourth order Runge-Kutta numerical method together with the Secant technique of root finding to determine their solutions. Graphical representation of the temperature, concentration and velocity fields and various other pertinent parameters such as Schmidt number Sc, Grashof number Gr, Eckert number Er for both mass and heat flow, and angle of inclination are presented and discussed. This study established that the flow field and other quantities of physical interest are significantly influenced by these parameters. In particular, it is found that the velocity increases with an increase in the thermal and solutal Grashof numbers. The velocity and concentration of the fluid decreases with an increase in the Schmidt number.

Keywords: Buoyancy driven flow, Inclined plate, MHD, shooting technique, Runge-Kutta method, heat and mass transfer, viscous dissipation

1 Introduction

Convective fluid motion is defined as the collective motion of particles of a fluid. There are two types of convection flow: forced convection and natural or free convection. Forced convection occurs when an external driving force causes the fluid to flow e.g. use of a fan, a pump or a blower. Free or natural convection is a self sustained flow driven by buoyancy effects due to density differences caused by temperature variation in fluid. The rate of heat transfer Q due to free convection is described by Newton's law of cooling.

$$Q = hA(T_w - T_\infty) = hA\Delta T \tag{1.1}$$

where h is the convection heat transfer coefficient, A is the surface area of plate, T_w is the temperature of the plate wall, T_∞ is the temperature of the surrounding and ΔT is the temperature difference.

There are two types of forces which generally act on a fluid. Body and surface forces. Body forces are proportional to the mass and hence density of the fluid and surface forces which are proportional to the surface area of the fluid. A good example of a body force is that due to gravity. If you consider a fluid of volume v, mass m and density ρ flowing past a hot vertical plate, the surrounding fluid will be called density ρ_∞ say. The body force on fluid will be mg where g is gravity force. The net force on fluid will be $(\rho - \rho_\infty)vg$. This is the buoyancy force driving the flow. During convective motion, both heat and mass of fluid are transferred. Convection heat transfer occurs at solid-liquid interface, solid-gas interface, liquid-gas interface, liquid-liquid interface and so on. Let us now derive a relationship between the temperature difference and the density difference found in the buoyancy force. Consider free convection flow bounded by a surface as shown in figure 1.1.

In this case $T_w > T_\infty$ i.e. the plate wall temperature is greater than the temperature inside the boundary layer. Now if a fluid flows past a solid, a fluid layer is formed adjacent to the boundary of the solid. This layer is called a boundary layer and strong viscous effects exist within this layer. It was first

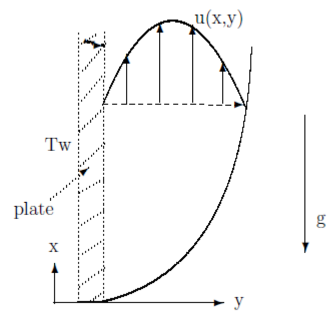

Figure 1.1: Convection flow

identified by Blasius in 1954. The momentum equation in this layer is then given by

$$\frac{Dv}{dt} = \frac{1}{\rho} f_B - \frac{1}{\rho}\nabla P + \frac{\mu}{\rho}\nabla^2 v \tag{1.2}$$

where $\vec{v}$ is the velocity given by $\vec{v} = u(x,y,z)i + v(x,y,z) + w(x,y,z)k$ in which $u(x,y,z), v(x,y,z)$ and $w(x,y,z)$ are the components of velocity in x, y and z directions respectively, $f_B = -\rho g$ is body force, P is the fluid pressure and μ is the dynamic viscosity of the fluid. Assuming steady 2-D incompressible flow, the x component of the momentum equation is given by

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} - g + \nu\frac{\partial^2 u}{\partial y^2} \tag{1.3}$$

where $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity. Note that the pressure gradient inside the boundary layer must balance the pressure gradient outside the boundary layer i.e.

$$\left(\frac{\partial P}{\partial x}\right)_{\text{inboundarylayer}} = -\rho g(\text{outsidetheboundarylayer})$$

so that 1.3 becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}(\rho_\infty g) - g + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\left(\frac{\rho_\infty - \rho}{\rho}\right) + \nu\frac{\partial^2 u}{\partial y^2}$$

$$\Rightarrow u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\left(\frac{\Delta\rho}{\rho}\right) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1.4}$$

Volumetric thermal expansion coefficient β is defined as

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

And in general,

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\left(\frac{\rho_\infty - \rho}{T_\infty - T}\right)$$

from which we find

$$\Delta\rho = \rho_\infty - \rho \approx \beta\rho(T - T_\infty) \tag{1.5}$$

using this expression in 1.4 we find

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1.6}$$

Convection is a major mode of heat and mass transfer in fluids and plays an important role in a wide range of engineering, scientific and industrial fields. Heat transfer by natural convection is utilized in a number of engineering practices for example cooling of equipment. Cooling of electronic circuit boards in computers is done by convection. To estimate surface temperatures of components mounted on a card or board one may approximate the surface as a flat plate. Convective heat and mass transfer is used in the design of heat exchangers, pumps etc. In science, heat transfer by natural convection has a bearing in the structure of the Earths atmosphere, its oceans and its mantle. Convective cells in the atmosphere can be seen as clouds, with stronger convection resulting in thunderstorms. Due and fog formation are a common feature on transparent surfaces. It creates a pattern that can cause blurred view over it. This blur is sometimes associated with safety concerns in airplanes and vehicles. This fogging or mist formation should be avoided by use of free convection over a flat surface. In industrial applications, MHD heat and mass transfer is used in metallurgical processes: cooling of many continuous strips or filaments, by drawing them through an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. Purification of molten metals from non metallic inclusions by application of a magnetic field. Glass production, furnace design are other examples.

Chen [5] performed an analysis to study the natural convection flow over a permeable inclined surface with variable wall temperature and concentration. The results show that the velocity is decreased in the presence of a magnetic field. Increasing the angle of inclination decreases the effect of buoyancy force. Heat transfer rate is increased when the Prandtl number is increased.

Finite difference Analysis of natural convection flow over a heated plate with different inclination and stability analysis was considered by Bengum et al. [1] and solved using Implicit finite difference method of Crank - Nicolson type.

Similarity Solutions for Hydromagnetic Free Convective Heat and Mass Transfer Flow along a Semi-Infinite Permeable Inclined Flat Plate with Heat Generation and Thermophoresis was studied by Sattar et al. [10].

Hydro magnetic incompressible viscous flow has many important engineering applications such as magneto hydrodynamic power generators and the cooling of reactors also its applications to problems in geophysics, astrophysics etc. The study of magnetohydrodynamics (MHD) plays an important role in agriculture, engineering and petroleum industries. The problem of free convection under the influence of a magnetic field has attracted the interest of many researchers in view of its applications in geophysics and astrophysics.

Manyonge et al. [3] examined On the Steady MHD Poiseuille Flow Between two Infinite Parallel Plates in an Inclined Magnetic Field where the governing equations were solved analytically and expressions for the fluid velocity obtained expressed in terms of Hartmann number.

Details of Effects of Variable Viscosity and Thermal Conductivity on MHD free Convection and Mass transfer Flow over an Inclined Vertical Surface in a Porous Medium with Heat Generation was investigated by S. Hazarika and G.C Hazarika [17] where the systems of ODEs were solved numerically by fourth order Runge-kutta method along with shooting technique.

On mhd heat and mass transfer over a moving vertical plate with a convective surface boundary condition was investigated by Makinde [14]. The similarity solution was used to transform the system of partial differential equations, describing the problem under consideration, into a boundary value problem of coupled ordinary differential equations, and an efficient numerical technique is implemented to solve the reduced system.

An analysis to study the effects of viscous dissipation and Joule heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of the combined effect of Hall and ion-slip currents for the case of power-law variation of the wall temperature was presented by Emad and Mohamed [7]. The governing differential equations are transformed by introducing proper non-similarity variables and solved numerically.

Singh [2] studied Mhd free convection and mass transfer flow with hall current, viscous dissipation, joule heating and thermal diffusion.

Kinyanjui et al.[18] performed Buoyancy effects of thermal and mass diffusion on mhd natural convection past finite vertical, porous at plate. The problem has been solved for velocity, temperature and concentration profiles. The equations governing the flow are solved numerically using finite difference method for various values of Grashof parameter ranging from 0 to -1. It was noted that a decrease in Grashof parameter leads to an increase in primary, secondary, temperature and concentration profile.

Combined heat and mass transfer problems of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid. Alam et al [11], studied on heat and mass transfer in MHD free convection flow over an inclined plate with Hall current, the governing PDEs were transformed using similarity transformations and solved numerically using Runge-Kutta fourth-fifth order with the help of symbolic software.

Singh [16], investigated heat and mass Transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium.

MHD Free Convective Heat and Mass Transfer Flow Past an Inclined Surface with Heat Generation was studied by Satter et al. [12]. The governing partial differential equations are reduced to a system of ordinary differential equations by introducing similarity transformations. The non-linear similarity equations are solved numerically by applying the Nachtsheim-Swigert shooting iteration technique together with a sixth order Runge-Kutta integration scheme.

Gnaneswara [8] analyzed Scaling transformation for Heat and Mass transfer effects on steady MHD free convection dissipative flow past an inclined porous surface.

Sivasankaran et al. [19], analyzed lie group analysis of natural convection heat and mass transfer in an inclined surface. It is found that the velocity increases with an increase in the thermal and solutal Grashof numbers. The velocity and concentration of the fluid decreases with an increase in the Schmidt number.

Numerous authors have studied MHD free convection flow with some extended effects along a vertical or horizontal plates. However, MHD free convection flow with some extended effects along an inclined plate has received inadequate attention since in many natural convection flows, the thermal input occurs at a surface that is itself curved or inclined with respect to the direction of the gravity field, therefore, it is for this fact that this study considered heat and mass transfer characteristics phenomenon on MHD free convection steady flow of an incompressible, electrically conducting fluid over an inclined heated infinite plate with varied inclination angle under the influence of an applied uniform magnetic field and combined effect of double diffusive, where dissipation and thermal diffusion taken into account with periodically varying surface temperature, when the temperature of the plate oscillates periodically about a constant mean temperature.

2 Mathematical Formulation

Let us consider steady two-dimensional laminar flow of a viscous, incompressible, electrically conducting fluid moving past a fixed inclined semi-infinite plate surface. The motion is in the presence of a uniform magnetic field of intensity B_0 applied normal to the plate surface. Assume the x axis of a

Cartesian coordinate system (x, y) is directed along the plate and the y axis is perpendicular to the plate surface. The origin of the coordinate system is taken to be the leading edge of the plate. The acceleration due to gravity g is taken to be acting vertically downwards. The plate surface is inclined to the vertical direction by an angle γ. The physical model and geometrical coordinate system are as shown below.

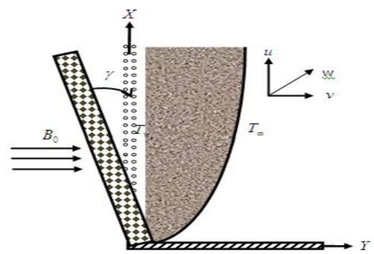

Figure 2.1: Physical configuration & coordinate system

We assume that the fluid property variations due to temperature and chemical species concentration are limited to fluid density. In addition, there is no applied electric field and all of the Hall effects and Joule heating are neglected. Since the magnetic Reynolds number is very small for most fluids used in industrial applications, we assume that the induced magnetic field is negligible[2]. Further, we shall neglect the Soret and Dufour effects as in [1] since we assume that the fluid under consideration has very small concentration of diffusing species in comparison to other chemical species and the concentration of species far from the plate wall, i.e. C_∞ is infinitesimally small. Let u and v be the velocity components in the x and y axes directions respectively. Under the Boussinesqs approximation within the boundary layer, the steady, laminar, two-dimensional boundary layer flow under consideration is governed by the equations of continuity, momentum, energy and species concentration respectively as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)\cos\gamma +$$

$$g\beta^*(C - C_\infty)\cos\gamma - \frac{\sigma_c B_0^2}{\rho}u \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} \tag{2.4}$$

where T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the uniform flow far away from the plate, β and β^* are thermal and concentration expansion coefficients respectively, ν is the kinematic viscosity, g is the gravitational acceleration, C_∞ - the species concentration in the fluid far away from the plate, C_p - specific heat at constant pressure, B_0 - the magnetic induction, α is thermal diffusivity, D_m is the chemical species diffusivity coefficient, σ_c is the electrical conductivity, ρ the fluid density, with the boundary conditions;

$$u = 0,\ T = T_\infty,\ C = C_\infty \text{ for all y, t} \leq 0$$

$$u = 0,\ T = T_w + \in (T_w - T_\infty)\cos\omega t,\ C = C_w \quad at$$

$$y = 0, t > 0$$

$$u \to 0,\ \ T \to T_\infty,\ \ C \to C_\infty\ as\ y \to \infty, t > 0$$

where T_w is the wall plate temperature and C_w is the chemical species concentration at the plate surface.

3 Method of Solution

The equations (2.2) to (2.4) are coupled, non linear partial differential equations and hence analytical solution is not possible. Therefore numerical technique is employed to obtain the required solution. Numerical computations are greatly facilitated by non dimensionalization of the equations. Proceeding with the analysis, we introduce the following similarity transformations and dimensionless variables which will convert the partial differential equations from two independent variables (x, y) to a system of coupled, non linear ordinary differential equations in a single variable (η) i.e. coordinate normal to the plate.

We now introduce a two-dimensional stream function $\psi(x, y)$ defined by $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$ so that continuity equation is automatically satisfied. In order to obtain a similarity solution of the problem we introduce the following non dimensional variables:

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}},\ \ \psi(x, y) = \sqrt{\nu U_\infty x} f(\eta),\ \ u = U_\infty f'(\eta),$$

$$v = \frac{1}{2}\sqrt{\frac{\nu U_\infty}{x}}(\eta f'(\eta) - f), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{3.1}$$

where η is a similarity variable, $\theta(\eta)$ and $\phi(\eta)$ are the dimensionless temperature and concentration respectively, U_∞ is the velocity of the fluid far away from the plate. Now substituting equation (3.1) in equations (2.2)- (2.4) we obtain:

$$f''' + \frac{1}{2}ff'' + \frac{1}{2}(f')^2 + \theta Gr\cos\gamma + \phi Gc\cos\gamma - Mf' = 0 \tag{3.2}$$

$$\theta'' + \frac{1}{2}Prf\theta' + PrE_c(f'')^2 = 0 \tag{3.3}$$

$$\phi'' + \frac{1}{2}Scf\phi' = 0 \tag{3.4}$$

where the prime symbol denotes differentiation with respect to η and

$$Gr = \frac{g\beta x(T_w - T_\infty)}{U_\infty^2}, \quad Gc = \frac{g\beta^* x(C_w - C_\infty)}{U_\infty^2}$$

$$Pr = \frac{v}{\alpha}, \quad E_c = \frac{U_\infty^2}{C_p(T_w - T_\infty)}, \quad M = \frac{\sigma_c B_0^2 x}{\rho U_\infty},$$

$$Sc = \frac{v}{D_m}, \quad \alpha = \frac{k}{\rho C_p}$$

in which Gr is the local thermal Grashof number, Gc is the solutal or local concentration Grashof number, Sc is the Schimdt number and Pr is the Prandtl number, E_c is the Erkert number and α is the thermal diffusivity. The corresponding initial and boundary conditions take the form :

$$u = 0, \quad \theta = 0, \quad C = 0 \;\; for \;\; all \;\; y, t \leq 0$$

$$u = 0, \;\; \theta = 1 + \in \cos\omega t, \quad C = 1 \;\; at \;\; y = 0, t > 0$$

$$u \to 0, \quad \theta \to 0, \quad C \to 0 \;\; as \;\; y \to \infty, t > 0$$

where ωt is phase angle.

4 Numerical Method

The similarity transformation converts the non-linear partial differential equations (2.2 - 2.4) into ordinary differential equations given by the set (3.2 3.4) which are solved numerically using a shooting method, a technique that converts the boundary value ordinary differential equations into a set of first order initial value ordinary differential equations with Secant iteration. The resulting system is solved by the fourth-order Runge-Kutta method implemented in Mathematica.

5 Results and Discussion

As a result of the numerical calculations, the dimensionless velocity, temperature and concentration distributions for the flow under consideration are obtained and their behaviour have been discussed for variations in the governing parameters viz., the thermal Grashof number Gr , solutal Grashof number Gc , magnetic field parameter M, angle of inclination γ, Prandtl number Pr, Eckert number Ec and Schmidt number Sc.

The influence of the thermal Grashof number on the velocity is presented in figure 5.1. The thermal Grashof number Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermo buoyancy force. Here, the positive values of Gr correspond to cooling of the plate. Also, as Gr increases, the fluid velocity increases, reaching its peak value within the boundary layer and then decreases monotonically to the free stream zero value far away from the plate surface satisfying the far field boundary condition.

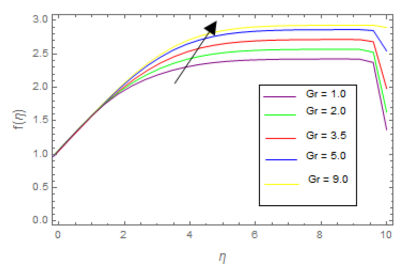

Figure 5.1: Velocity profiles for different values of Gr

It is interesting however to note that the velocity boundary layer thickness increases while the thermal boundary layer thickness decreases with an increase in the value of thermal Grashof number (Gr) figure 5.2.

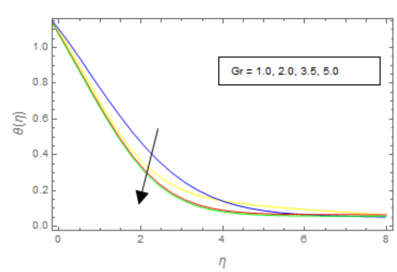

Figure 5.2: Temperature profiles for different values of Gr

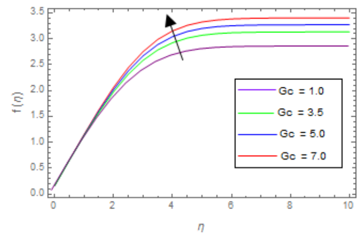

Figure 5.3: Velocity profiles for different values of Gc

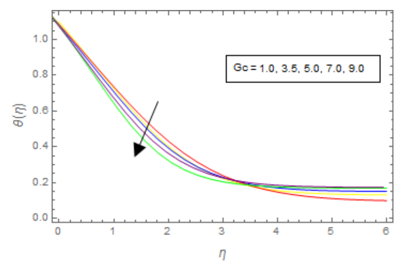

Figure 5.4: Temperature profiles for different values of Gc

Figure 5.3 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number Gc, while all other parameters are kept at some fixed values. The solutal Grashof number Gc defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity
increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. Moreover, an increase in the intensity of buoyancy forces (Gc), causes a decrease in the fluid temperature leading to a decaying thermal boundary layer thickness.

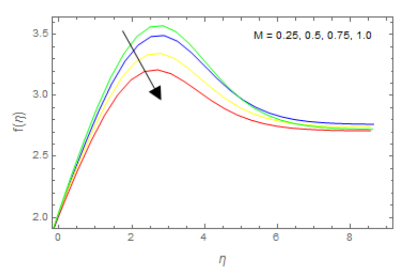

Figure 5.5: Velocity profiles for different values of M

For various values of the magnetic parameter M, the velocity profiles are plotted in Figure 5.5. An increase in magnetic field parameter, M, is observed to strongly reduce the velocity in the regime. Maximum velocity corresponds to

$M = 0$ i.e. electrically non conducting heat and mass transfer. Physically, it is true due to the fact that the application of a transverse magnetic field to an electrically conducting fluid gives rise to a body force known as a Lorentz hydromagnetic drag which acts in the tangential direction. This force, $-(M)f'$, impedes the flow and reduces velocities i.e. decreases the hydrodynamic boundary layer thickness.

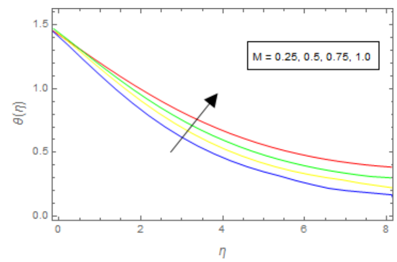

Figure 5.6: Temperature profiles for different values of M

From figure 5.6, we see that the temperature profiles increase with the increase of the magnetic field parameter. This result qualitatively agrees with the expectations, since the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow and slow down its motion in the boundary layer region. This, in turn, reduces the rate of heat convection in the flow i.e. which implies that the applied magnetic field tends to heat the fluid and thus reduces the heat transfer from the wall which appears in increasing the flow temperature and thermal boundary layer thickness also boosted with increasing M values.

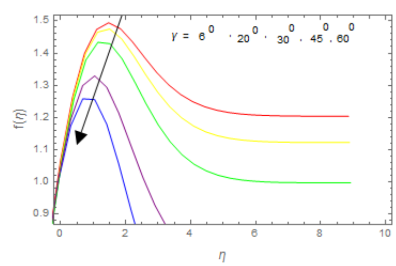

Figure 5.7: Velocity profiles for different values of γ

Figure 5.7 shows the effect of angle of inclination to the vertical direction on the velocity profiles. From this figure we observe that the velocity is decreased by increasing the angle of inclination γ. The fluid has higher velocity when the surface is vertical ($\gamma = 0$) than when inclined because of the fact that the buoyancy effect decreases due to gravity components ($g \cos \gamma$), as the plate is inclined.

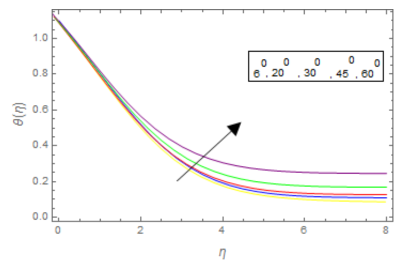

Figure 5.8: Temperature profiles for different γ

We observe in figure 5.8 that both the thermal and concentration boundary layer thickness increase as the angle of inclination increases.

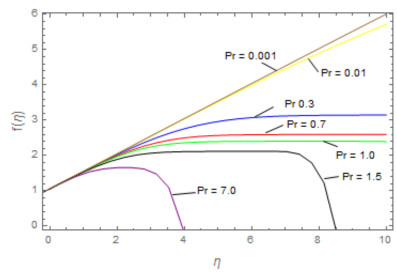

Figure 5.9: Velocity profiles for different values of Pr

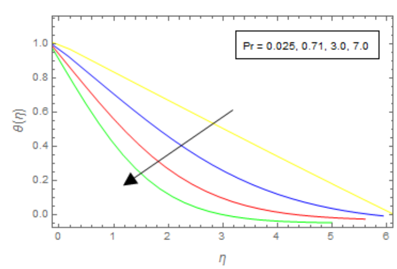

Figure 5.10: Temperature profiles for different values of Pr

Figure 5.9 and 5.10 illustrate the velocity and temperature profiles for different values of the Prandtl number Pr. The Prandtl number defines the ratio of momentum diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Fig. 5.9). From figure 5.10, it is observed that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of Pr. i.e. velocity for $Pr = 0.71$ is higher than that of $Pr = 7$. Physically, it is possible because fluids with high Prandtl number have high viscosity and hence move slowly.

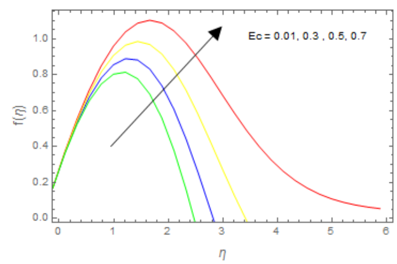

Figure 5.11: Velocity profiles for different values of Eckert (Ec)

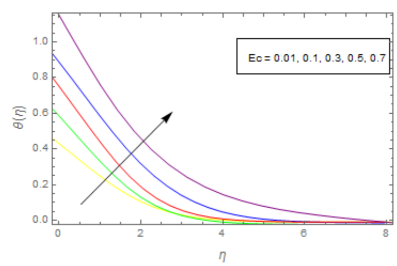

Figure 5.12: Temperature profiles for different values of Eckert (Ec)

The effect of the viscous dissipation parameter i.e., the Eckert number Ec on the velocity and temperature are shown in Figs 5.11 and 5.12 respectively. The Eckert number expresses the relationship between the kinetic energy of the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the velocity as well as temperature which is evidenced in the above figures.

The influences of the Schmidt number Sc on the velocity and concentration profiles are plotted in Figure 5.13 and 5.14. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles

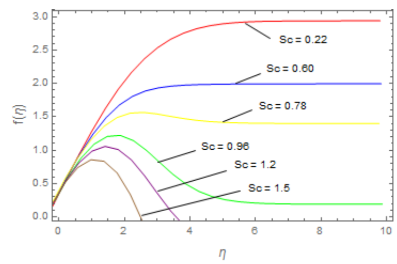

Figure 5.13: Velocity profiles for different values of Schmidt (Sc) Number

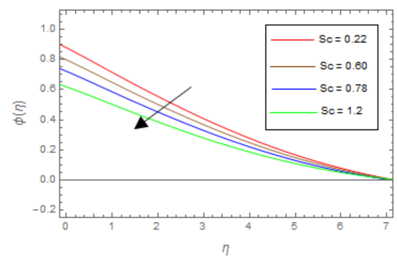

Figure 5.14: Concentration profiles for different values of Sc

are accompanied by simultaneous reductions in the velocity and concentration boundary layers. It is worth to mention that for hydrogen ($Sc = 0.22$) the velocity profiles is much higher than that of other Sc.

6 Conclusions

In many practical applications, the characteristics involved, such as the heat transfer rate at the surface are vital since they influence the quality of the final product. The present work, helps us in understanding numerically as well as physically free convection flow in an inclined infinite flat plate in the presence of MHD where viscous dissipation has been employed. The effect of inclination and variation of other controlling physical parameters have been studied and their effects presented. Thus, applications of effects of some parameters are recommended for cooling and heating in industrial processes.
Based on the results the effects of increasing values of the physical parameters which had significant effect on velocity, temperature and concentration profiles were as follows;

- In natural convection flow velocity is sufficiently small, the Prandtl number has no significant effect on concentration. However, it is observed that increase in Prandtl number (Pr) leads to a decrease in velocity and temperature. This result is in conformity with the known and observed facts that in liquid metals ($Pr < 1$) the heat diffuses faster as compared to the lubricant oils ($Pr > 1$).

- We observed that the fluid (air) velocity decreases for an increase in angle of inclination γ. The fluid has higher velocity when the surface is vertical $\gamma = 0$ than when inclined because the convective flow under consideration takes place due to the interaction of gravity and density differences and in the inclined position the effective gravity force is less than what it is when the plate vertical. On the other hand, both the temperature and concentration profiles increase with an increase of γ.

- It is to be noted that an increase in the magnetic field has significant effect on the velocity, temperature and concentration profiles. It leads to a rise in temperature and concentration at a slow rate in comparison to the velocity profiles. In the presence of the magnetic field, the velocity boundary layer is thinner than the temperature and the concentration boundary layer. So magnetic field can effectively be used to control the flow characteristics and heat transfer.

- From the numerical results, the positive values of thermal Grashof number $Gr > 0$ is utilised in our computations. This corresponds to the cooling problem with respect to application. The cooling problem is often encountered in engineering applications for example in the cooling of electronic components and nuclear reactors. It was found that when thermal and solutal Grashof numbers were increased, the thermal and concentration buoyancy effects were enhanced and thus, the fluid velocity increased.

- It was noticed that an increase in Eckert number enhanced the velocity and temperature profiles but a decrease in concentration was observed when Eckert number was increased.

- An increase in Schmidt number results in lowering the concentration and velocity while temperature of the fluid increases. Therefore, Schmidt number has greater effect on concentration profiles than the velocity and temperature profiles. So, we can dominate the rate of mass transfer with the help of the Schmidt number.

Recommendations

- It is therefore recommended that in applying the technique of inclination to enhance cooling of materials in industrial processes, the range of the cooling angle should be considered.

- The Schmidt number which enhances mass diffusivity should be considered in processes involving fluid transportation.

- The viscous dissipation parameter had an integral effect in increasing the temperature in the boundary layer and therefore should be considered in the design of heating systems.

- An attempt should be made to solve this problem using other numerical techniques and compare results.

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