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Mathematical Modelling of Wind Turbine in a Wind Energy Conversion System: Power Coefficient Analysis

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Abstract

The world is increasingly going green in its energy use. Wind power is a green renewable source of energy that can compete effectively with fossil fuel as a generator of power in the electricity market. For this effective competion, the production cost must be comparable to that of fossil fuels or other sources of energy. The initial capital investment in wind power goes to machine and the supporting infrastructure. Any factors that lead to decrease in cost of energy such as turbine design, construction and operation are key to making wind power competitive as an alternative source of energy. A mathematical model of wind turbine is essential in the understanding of the behaviour of the wind turbine over its region of operation because it allows for the development of comprehensive control algorithms that aid in optimal operation of a wind turbine. Modelling enables control of wind turbine's performance. This paper attempts to address part or whole of these general objectives of wind turbine modelling through examination of power coefficient parameter. Model results will be beneficial to designers and researchers of new generation turbines who can utilize the information to optimize the design of turbines and minimize generation costs leading to decrease in cost of wind energy and hence, making it an economically viable alternative source of energy.

Mathematics Subject Classification: 65C20

Keywords: Wind velocity, Turbine power, Power coefficient, Tip speed ratio, Generator, Grid

1 Introduction

At this moment in time, the world is going the way of green energy(renewable energies) in its energy consumption. Wind energy or wind power describe the process by which wind is used to generate mechanical or electric power. Use of wind energy for electricity generation purposes is becoming an increasingly attractive energy source partly due to the increase in energy demand worldwide and environmental concerns. Burning of fossil fuels emit gases such as carbon dioxide into the atmosphere that lead to global warming. Wind energy does not rely on fossil fuels for energy generation. A typical wind energy conversion system consists of three major devices making up a wind turbine that convert wind energy to electric energy. The first device is the rotor which consists of two or three fibre glass blades joined to a hub that contains hydraulic motors that change each blade according to prevailing wind conditions so that the turbine can operate efficiently at varying wind speeds. The nacelle is a large housing behind the rotor that houses the drive shaft, gearbox, transformer and generator. Nacelle is usually mounted over a yaw gear which turns it and the rotor so that the wind is normal to the rotor plane all the time for maximum tapping of energy from the wind. The tower supports the rotor and the nacelle.

The kinetic energy in the wind is converted into mechanical energy by the turbine by way of shaft and gearbox arrangement because of the different operating speed ranges of the wind turbine rotor and generator. The generator converts this mechanical energy into electrical energy. The mechanical power obtained can be used to perform important tasks such as grinding of grain or pumping of water. The electricity generated can be used in human daily activities. It can be used to power homes, schools, hospital, industries, businesses etc.

The major wind energy system components that lend themselves to modelling can be grouped as follows: (i) the wind model, (ii) the turbine model, (iii) the shaft and gearbox model, (iv) the generator model and (v) the control system model. In this paper we shall confine ourselves to the study of the turbine model.

1.1 Turbine Model

A wind turbine consists of a rotor mounted to a nacelle and a tower with two or more blades mechanically connected to an electric generator. The gearbox in the mechanical assembly transforms slower rotational speeds of the wind turbine to higher rotational speeds on the electric generator. The rotation of the electric generator's shaft generates electricity whose out put is maintained by a control system. There are two types of design models for wind turbines. The classification is made on the basis of their axis in which the turbines rotate: Horizontal Axis Wind Turbine (HAWT) and Vertical Axis Wind Turbine (VAWT). The VAWT is also called Darrieus rotor named after its inventor[6]. HAWT have the ability to collect maximum amount of wind energy for time of day and season and their blades can be adjusted to avoid high wind storm. Wind turbines operate in two modes namely constant or variable speed. For a constant speed turbine, the rotor turns at constant angular speed regardless of wind variations. One advantage of this mode is that it eliminates expensive power electronics such as inverters and converters. Its disadvantage however, is that it constrains rotor speed so that the turbine cannot operate at its peak efficiency in all wind speeds. For this reason a constant wind speed turbine produces less energy at low wind speeds than does a variable wind speed turbine which is designed to operate at a rotor speed proportional to the wind speed below its rated wind speed[1].

The output power or torque of a wind turbine is determined by several factors. Among them are (i) turbine speed, (ii) rotor blade tilt, (iii) rotor blade pitch angle (iv) size and shape of turbine, (v) area of turbine, (vi) rotor geometry whether it is a HAWT or a VAWT, (vii) and wind speed. A relationship between the output power and the various variables constitute the mathematical model of the wind turbine. A mathematical model of wind turbine is essential in the understanding of the behaviour of the wind turbine over its region of operation and also modelling enables control of wind turbine's performance. This paper attempts to address part or whole of these general objectives of wind turbine modelling.

1.2 Mathematical Formulation of Turbine Model

Under constant acceleration a, the kinetic energy E of an object having mass m and velocity v is equal to the work done W in displacing that object from rest to a distance s under a force F, i.e. $E = W = Fs$. According to Newton's second law of motion

$$F = ma \tag{1}$$

thus, the kinetic energy becomes

$$E = mas \tag{2}$$

From kinematics of solid motion, $v^2 = u^2 + 2as$ where u is the initial velocity of the object. This implies that $a = \frac{v^2-u^2}{2s}$. Assuming the initial velocity of the object is zero, we have that $a = \frac{v^2}{2s}$. Hence from equation (2) we have that

$$E = \frac{1}{2}mv^2 \tag{3}$$

This kinetic energy formulation is based on the fact that the mass of the solid is a constant. However, if we consider wind (air in motion) as a fluid, both density and velocity can change and hence no constant mass. For this reason Reccab et. al[5] formulate the kinetic energy law with a factor of $\frac{2}{3}$ instead of $\frac{1}{2}$. In this paper we shall assume that the density of air does not vary considerably even with variation in altitude or temperature and use the kinetic energy law in the form of equation (3). Hence the kinetic energy(in joules) in air of mass m moving with velocity v_w(wind) can be calculated from equation (3) above. The power P in the wind is given by the rate of change of kinetic energy, i.e.

$$P = \frac{dE}{dt} = \frac{1}{2}\frac{dm}{dt}v_w^2 \tag{4}$$

But mass flow rate $\frac{dm}{dt}$ is given by $\frac{dm}{dt} = \rho A v_w$ where A is the area through which the wind in this case is flowing and ρ is the density of air. With this expression, equation (4) becomes

$$P = \frac{1}{2}\rho A v_w^3 \tag{5}$$

The actual mechanical power P_w extracted by the rotor blades in watts is the difference between the upstream and the downstream wind powers[1], i.e.

$$P_w = \frac{1}{2}\rho A v_w(v_u^2 - v_d^2) \tag{6}$$

where v_u is the upstream wind velocity at the entrance of the rotor blades in m/s and v_d is the downstream wind velocity at the exit of the rotor blades in m/s. We shall see later that these two velocities give rise to the blade tip speed ratio. Now from the mass flow rate, we may write

$$\rho A v_w = \frac{\rho A(v_u + v_d)}{2} \tag{7}$$

v_w being the average of the velocities at the entry and exit of rotor blades of turbine. With this expression, equation(6) becomes $P_w = \frac{1}{2}\rho A(v_u^2 - v_d^2)\frac{(v_u+v_d)}{2}$ which may be simplified as follows:

$$\begin{aligned} P_w &= \frac{1}{2}\left[\rho A\left\{\frac{v_u}{2}(v_u^2 - v_d^2) + \frac{v_d}{2}(v_u^2 - v_d^2)\right\}\right] \\ &= \frac{1}{2}\left[\rho A\left\{\frac{v_u^3}{2} - \frac{v_u v_d^2}{2} + \frac{v_d v_u^2}{2} - \frac{v_d^3}{2}\right\}\right] \\ &= \frac{1}{2}\left[\rho A v_u^3\left\{\frac{1 - (\frac{v_d}{v_u})^2 + (\frac{v_d}{v_u}) - (\frac{v_d}{v_u})^3}{2}\right\}\right] \end{aligned}$$

or

$$P_w = \frac{1}{2}\rho A V_u^3 C_p \tag{8}$$

where $C_p = \frac{1-(\frac{v_d}{v_u})^2+(\frac{v_d}{v_u})-(\frac{v_d}{v_u})^3}{2}$ or

$$C_p = \frac{(1 + \frac{v_d}{v_u})(1 - (\frac{v_d}{v_u})^2)}{2} \tag{9}$$

The expression for C_p in equation (9) is the fraction of upstream wind power captured by the rotor blades. C_p is often called the Betz limit after the Germany physicist Albert Betz who worked it out in 1919. Other names for this quantity are the power coefficient of the rotor or rotor efficiency. The power coefficient is not a static value. It varies with tip speed ratio of the wind turbine. Let λ represent the ratio of wind speed v_d downstream to wind speed v_u upstream of the turbine, i.e.

$$\lambda = \frac{v_d}{v_u} \tag{10}$$

or

$$\lambda = \frac{\text{blade tip speed}}{\text{wind speed}} \tag{11}$$

λ is called the tip speed ratio of the wind turbine. The blade tip speed in metres per second can be calculated from the rotational speed of the turbine and the length of the blades used in the turbine, i.e.

$$\text{blade tip speed} = \frac{\text{angular speed of turbine}(\omega) \times R}{\text{wind speed}} \tag{12}$$

where R is the radius of the turbine and ω is measured in radian per second. Substitution of equation (10) into equation (9) leads to

$$C_p = \frac{(1+\lambda)(1-\lambda^2)}{2} \tag{13}$$

Differentiate C_p with respect to λ and equate to zero to find value of λ that makes C_p a maximum, i.e. $\frac{dC_p}{d\lambda} = \frac{(1+\lambda).(-2\lambda)+(1-\lambda^2).1}{2} = 0$ yielding $\lambda = -1$ or $\lambda = \frac{1}{3}$. Now $\lambda = \frac{1}{3}$ makes the value of C_p a maximum. This maximum value is $\frac{16}{27}$. Thus the Betz limit says that no wind turbine can convert more than $\frac{16}{27}$ (59.3%) of the kinetic energy of the wind into mechanical energy turning a rotor, i.e $C_{pmax} = 0.59$. Wind turbines cannot operate at this maximum limit though. The real world is well below the Betz limit with values of $0.35 - 0.45$ common even in best designed wind turbines.

If the rotor of a wind turbine turns too slowly most of the wind will pass through the openings between blades with little power extraction. If on the other hand the rotor turns too fast, the rotating blades act as a solid wall obstructing the wind flow again reducing the power extraction. The turbines must be designed to operate at their optimal wind tip speed ratio λ in order to extract as much power as possible from the wind stream. Theoretically the higher the λ the better in terms of efficient operation of the generator. There are disadvantages however. High λ causes erosion of leading edges of the blades due to impact of dust or sand particles found in the air. This would require use of special erosion resistant coating material that may increase the cost of energy. Higher λ also leads to noise generation, vibration, reduced rotor efficiency due to drag and tip losses and excessive rotor speeds can lead to turbine failure.

Other factors that impede complete energy conversion in a complete turbine system are things such as gearbox, bearings, number and shape of blades etc. Only $10 - 30\%$ of the power of the wind is ever actually converted into usable electricity.

Air density ρ is another flow input quantity at the rotor system. ρ is a function of both air pressure and temperature. When air pressure increases ρ increases. When air temperature decreases ρ increases. This is in accordance with the equation of state

$$P = \rho R T \tag{14}$$

where R is the gas constant. Both temperature and pressure decrease with increasing elevation. Hence site location is important as elevation has major effect on power generated as a result of air density variation. At atmospheric pressure, $P_{\text{atm}} = 14.7psi$, temperature is $T = 60^0 F$ and density is $\rho = 1.225 kg/m^3$. Temperature and pressure both vary with elevation. This affects the air density. [6] propose the following relation

$$\rho = \rho_0 e^{-\frac{0.297}{3048} H_m} \tag{15}$$

where H_m is site elevation in metres. At high elevations the air density corrections can be important.

1.3 Power Coefficient Analysis

Equation (8) relates the parameters that are required in power production by a wind turbine. The power coefficient C_p is the most important parameter in the case of power regulation[4]. It is a non-linear function whose value is unique to each turbine type and is a function of wind speed that the turbine is operating in. Each turbine manufacturer provides look up tables for C_p for operation purposes. Other than look up tables from turbine manufactures, models for power coefficient have been developed. For example [3] models C_p as a function of the tip speed ratio and the blade pitch angle θ in degrees as

$$C_p(\lambda, \theta) = C_1 (C_2 \frac{1}{\beta} - C_3 \beta \theta - C_4 \theta^x - C_5) e^{-C_6 \frac{1}{\beta}} \tag{16}$$

where the values of the coefficients $C_1 - C_6$ and x depend on turbine type. θ is defined as the angle between the plane of rotation and the blade cross section chord. For a particular turbine type $C_1 = 0.5, C_2 = 116, C_3 = 0.4, C_4 = 0, C_5 = 5, C_6 = 21$ and β is defined by

$$\frac{1}{\beta} = \frac{1}{\lambda + 0.08\theta} - \frac{0.035}{1 + \theta^3} \tag{17}$$

Anderson and Bose [3] suggested the following empirical relation for C_p

$$C_p = \frac{1}{2}(\lambda - 0.022\theta^2 - 5.6) e^{-0.17\lambda} \tag{18}$$

where θ is the pitch angle of the blade in degrees, λ is the tip speed ratio of the turbine defined by $\lambda = \frac{v_w(mph)}{\omega_b(rads^{-1})}$ where ω_b is the turbine angular speed.

1.4 Modelling and Control of the Power Output

The power output of a turbine as we have mentioned is determined by the area of the rotor blades, wind speed and the power coefficient. The output power of the turbine can be varied by changing the area and flow conditions at the rotor system and this forms the basis of the control system. C_p is achieved at a particular λ which is specific to the design of the turbine.
Hence the model turbine consists of equations (5), power in the wind, equation (8), power captured by the turbine, equation(10), he tip speed ratio of the turbine and the power coefficient equation (16). Control of out put of wind energy lies in a number of parameters. The rotor area and flow conditions at the rotor system(v_w, ρ, C_p), the rotor torque and pitch angle control. Fixed speed stall-regulated turbines have no options for control input. However, variable speed wind turbines use generator torque to control and optimize power output. They also use pitch control to control the output power above their rated wind speed. Next we use MATLAB to simulate and perform an analysis of the variation of C_p against λ. With varying θ we obtain curves called performance curves for a given 'best' turbine and simulate its 'best' operation range. For our simulated turbine, we use the constants suggested in [3] with variation in the term $\frac{1}{1+.08\theta}$ becoming $\frac{1}{1+.08\theta+.0001}$. This additional reasonable constant avoids infinity values when $\lambda = 0$. The C_p is plotted for values of $\theta = 0^0$, $\theta = 3^0$,$\theta = 6^0$, $\theta = 10^0$, $\theta = 15^0$ and $\theta = 24^0$. This is depicted in figure 1 below. From the figure, it is clear that pitching the blades of the turbine reduces C_p from about 40% when the pitch angle is 0 to about 10% when the pitch angle increases to 3^0. This factor is good because it controls power output of a variables wind speed turbine (like the one we are considering) when the wind speed is above the rated one. Another turbine design model has the constant $C_4 = -0.5$. This increases the value of the efficiency C_p and the range of the tip speed ratio, however, at first pitching of 3^0, the efficiency of the turbine drastically reduces to about 10% as shown in figure 2.

In conclusion we have modelled a variable speed turbine with a particular consideration of some the design parameters. The turbine operation exhibits a reasonable range of tip speed ratio and high efficiency.

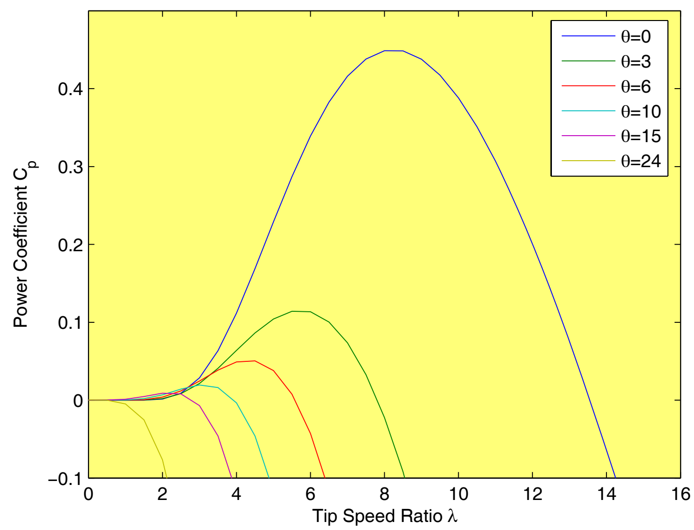

Figure 1: Simulated Power Coefficient

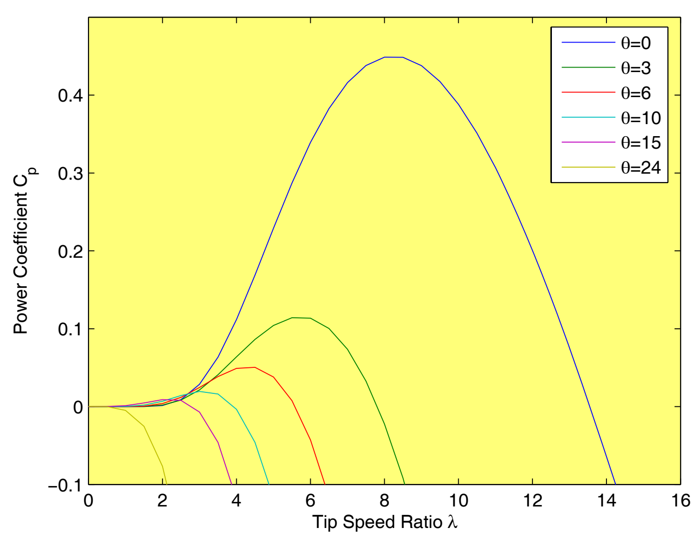

Figure 2: Simulated Power Coefficient(with variation of constant C_4)

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