AN ANALYSIS OF PATIENT FLOW USING MARKOV CHAINS: A CASE OF KAPSABET COUNTY REFERRAL HOSPITAL

BY

MERARY KIPKOGEI

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SCHOOL OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

MASENO UNIVERSITY

Declaration

This project is my own work and has not been presented for a degree award in any other institution.

MERARY KIPKOGEI

MSC/MAT/00039/17

Signature:....................................Date:................................

This project has been submitted for examination with my approval as the university supervisor.

DR. EDGAR A. OUKO OTUMBA PhD.

Department of Statistics and Actuarial Science

Maseno University

Signature:......................................Date:...............................

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Dedication

To my my loving parents, Mr and Mrs Misoi, my siblings; Geoffrey, Ruth, Sammy and Mercy thanks for your support.

Abstract

Hospital is an essential welfare of society. It provides management of illnesses through treatment and prevention interventions by medical and health professionals. Due to growing population and rise in chronic diseases, there is an increased demand for health care services. This causes congestion and overcrowding in most hospitals. Hospital overcrowding is a major problem to patients, hospital administration and to the general health workers. Hospitals in many jurisdictions struggle to reduce congestion and improve the patient flow across the continuum of care.

In this project report, we have developed an objective patient flow assessment through an analysis of Markov chains using weekly data from Kapsabet county referral hospital to check on the patient's flow at the hospital.

We used weekly data to construct transition matrices for each day in a week to portray the weekly routine amount of the patients in the hospital using the states; high, medium,low and very low. Steady state transition matrices were also computed for each day of the week to reflect the future flow for each week.

It was found that the patient flow had some pattern observed through the steady states. The probability of patient flow being high tends to be up on Mondays with probability of 0.57, medium on Tuesdays to Thursdays with probabilities of being high ranging from 0.36 on Tuesdays and 0.3 on Thursdays, on Fridays is when the steady states of being high starting to decrease upto Sunday with steady state probabilities of 0.22 on Friday, 0.17 on Saturday and 0.12 on Sunday.

Through the analysis of patient flow using Markov chains, we have identified some pattern of how the patient flow throughout the week. Generally through this study, the patient flow congestion can be easily understood, handled and hence controlled

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CHAPTER ONE

INTRODUCTION

Hospital is an essential welfare of society. It provides management of illnesses through treatment and prevention interventions by medical and health professionals [1]. Due to the growing population, there is rise in chronic diseases hence causing an increased demand for healthcare services in country's hospitals [2]. Currently, most hospitals are operating at or to almost full capacity most of the time. Bed availability is randomly distributed because of the random nature of the patients' arrivals and duration of treatments. Due to this, episodes of congestion and overcrowding can occur which lead to wide range of consequences such as long waiting times, lower attention from practitioners and crowded facilities [3].

Hospital overcrowding is a headache problem to patients, hospital administrators and to general health workers. Patients in public hospitals suffer from overcrowding and its effects. [4]Stated that overcrowding in emergency and inpatient units causes a series of negative effects such as medical errors, poor patients' outcome and patient dissatisfaction. On the other hand [5]observed that overcrowding in emergency department caused delayed treatments, increased mortality, financial losses and prolonged waiting for the patients and this causes formation of long queues.

When waiting rooms are full, the patients may leave without receiving the care services or the care facilities may temporarily stop accepting new arrivals and because of this, congestion in hospitals is almost becoming considerable hindrance to quality of care and staff morale.

Overcrowding is a continuous problem especially in government hospitals. Emergency rooms crowding, patient beds extending to hospital corridors, mothers sharing beds in obstetrics wards. These are not rare occurrences in government hospitals. Congestion has been a concern through the years and it has continually persists despite the awareness of the problem. The problem even worsen in times of disaster and calamities[6]. This has been the case at Kapsabet County Referral Hospital

Patient flow is primarily associated with hospitals, especially with backups and overcrowding in emergency departments and inefficient scheduling in surgical departments.
In light of these challenges, a need for a reform to our hospitals has become urgent to control crowding. In order to reduce overcrowding burden on provision of health care services, we therefore used a model that checks through the patient flow system. This was possibly achieved by using a predictive model.

1.1 STATEMENT OF THE PROBLEM

Patient flow represents the ability of health care to serve the patients quickly and efficiently as they move through the stages of care. Because of growing population, there is a rise in chronic diseases hence causing increased demand for health care services in hospitals. Currently, most hospitals are operating at or to almost full capacity most of the time. Due to this, episodes of congestion and overcrowding occur which lead to wide range of consequences such as long queues, long waiting times, lowering attention from practitioners and crowded facilities. Hospital overcrowding is a headache problem to patients, hospital administrators and to the general health workers. Congestion in hospitals is almost becoming considerable hindrance to quality of care and to staff morale. To maintain the quality of the services, there is need to evaluate where the patient flow process is being held up and what improvements need to be done to correct the congestion problem. In this project, we used Markov chains to model the patient flow at Kapsabet County Referral Hospital.

1.2 OBJECTIVES OF THE STUDY

1.2.1 General objectives

To apply Markov chain in analyzing patient flow in a hospital environment.

1.2.2 Specific objectives

1. To examine if the proposed model can be used to create transition matrices showing the flow of the patients in a hospital.

2. To analyze if the proposed model can be used to find steady state probabilities of the flow.

1.3 SIGNIFICANCE OF THE STUDY

The findings of this study is intended for hospital management who deal with patient flow directly and need to understand how hospital space is utilized.
Through the results of this study, the hospital managers will find it easier to do a better resource planning framework so as to meet the hospital huge numbers of patients they receive in their facilities.
This study will enable the hospital managers when to discharge the patients, when to schedule patients and booking for admissions.
The findings of this study will be used by the ministry of health in the counties and hospital managers to do proper facilitation of the resources that are required to reduce the congestion problem in the hospitals.

CHAPTER TWO

LITERATURE REVIEW

This chapter discusses the literature on overcrowding, patient flow and applications of Markov chains in health care problems.

More patients seeking care and few inpatients beds are available, hospitals grow crowded with admitted patients who could not be transitioned to inpatient care [7]. Many hospitals across the country are crowded. Nearly half of hospitals report operating at or above the capacity and 9 out of 10 hospitals report holding boarding admitted patients while they await inpatient bed [8].

Overcrowding is not limited to hospitals either within certain areas, overcrowding harms hospitals in academic, country and private hospitals alike; regardless of where they are in urban and rural areas [9]. With an increased capacity at all levels of health care delivery system, there has been an increased pressure for tighter financial management. Hospitals are faced with reduced flexibility and ability to accommodate the variation in demand [10]. Hospital overcrowding is a major problem for all types of health care organizations. Overcrowding has become so bad that more than six out of every ten hospitals across the country are operating at or over capacity [11].

When hospitals are overcrowded medics tend to reprioritize patients' needs. Typically medics will address the patients' higher level needs because there is neither time, space nor equipment to address the lower level needs [12]. Example is when a hospital is not at full capacity, a medic usually have time to provide patient education, explain written discharge instruction and answer any questions that the patient might have. This is to ensure that the patient is well informed of their illnesses and is aware of what she/he will do upon returning home from hospital. However, when the hospital is crowded the medic may not have time to give patients the written instructions and forgo the explanation. The patient's depth of understanding is then compromised and patient will probably end back in hospital for medical treatment [13].

2.1 Patient flow

Patient flow is a study of investigating on ways in which patients are transferred inside the health care system [14].[22]During the care process, the patients move through different routes in health care system. Different patient types choose various paths which some of the routes can be similar and rest differently.

Patient flow is significant element which influences healthcare service performance. Patient flow is ensuring that the patients receive the care quickly and efficiently as they pass through the stages of care when they need it without minimal delay. It is one of the greatest challenge facing healthcare today. Patient flow, effective movement of patients in healthcare process is an important indicator of effective and efficient hospital management. A well-organized patient flow can reduce delay of health care services[16]. Patient flow modeling is a quality improvement tool which can be used to help identify patient flow inefficiency at any type of healthcare facility and inform the area for intervention to help improve the care delivery processes[17].

Regarding to modeling of patient flow, [22] modeling the patient flow has significant benefits for hospital service administration by offering visions for enhancing resource development, capacity allocation and planning and appointment arrangements. This is the reason why the health care in many jurisdictions struggle to reduce congestion and improve patient flow across the continuum of the care process[18].

A patient pathway for a system is defined as the way a patient sequentially move from one department to another. The patients can move from one department to another and the patients can be discharged at any department or after moving through some depatments, a patient can die in any hospital department[21] as the patients move to receive the care process, the number of the patients changes with time.

In order to find some possible solutions for improving the patient flow, mathematical models for predicting the number of patients should be adopted for hospital planning.

[19]Analysis of patient flow should be performed with stochastic nature such as patient routing. Patient flow through all phases in a hospital system can enable us predict the number of patients in various phases at a given time in future[20]. Prediction is one good method where if done accurately, it can influence planning and guide the allocation of

resources to facilitate proper patient flow.

Patient flow for care process in hospitals are completed by different states which can be characterized by Markov chain. These models can be utilized by defining patients' number for the future for evaluating patients length of stay, time spent in each state,resource allocation and cost estimation for care process[22].

Analyzing patient flow in hospitals is important in providing early warning of crowding in these hospitals, monitoring and controlling the crowding. In this report we have used Markov chain theory as the mathematical model to analyze the patient flow .

2.2 Applications of Markov chains

Markov chains have been used in many real world applications including health care problems. There is rather extensive literature on Markov chain models applied to describe the stochastic dynamics of patient flow in health care.

[23]Formed a group of patients admitted to the same specialty with some arrival rates at the hospital, a distinct and independent semi Markov chain was processed for each of these groups. The authors developed the necessary formulation to estimate performance parameters.

[26]and[35]modeled semi Markov process, but they considered the flow of one group of patients. The study of [26]focused on modeling a hospital as being formed by incapacitated facility units, represented transition state in semi Markov process. Cote and stain went further and introduced the Erlang distribution for governing transition probability among the states.

[27]Applied proportion calculations in order to estimate the transition probabilities among hospital facilities represented by discrete time Markov chain. Through Markov chains he obtained performance parameters, taking the hospital as closed system comprised of recurrent states and unique groups of patients.

[28]Presented a model similar to that of Smallwood and Kao and proposed an iterative methodology for testing validity of Markovian characteristic assumptions.[29] Explained all types of patient flow including operative or clinical in any process such as; outpatient,

inpatient, surgical and emergency care. From an operative type various. Steps such as registering, consultation and medical tests. Various spots have possible diverse transitions to further states and even dissimilar patients' category have numerous probabilities of transitions between the states. Defining the probabilities should be performed according to examining the earlier information on patients' pathways[34].

[31]Introduced the Markov model with continuous time in order to analyze length of stay for older patients who are transported between their home and nurses' location.

[32]Introduced a different version of Markov model with discrete time period for ward admittance and capacity development in care system in which treatment is provided both in society and health centers. [33]Explained that patient flow structure can be modeled as Markov chain process as it specify the states and possible transitions between them. During the care process, the patients move through different routes in hospital. Different patient types choose various paths which some of the routes can be similar and rests differently. Various spots have possible diverse transitions to further states and even similar patients category have numerous probability transitions between states. Defining the probabilities should be performed according to examining earlier information on patients pathways.[34],[35]and[36] Used Markov chain models in order to model patient transfer in an intensive care unit in a hospital in Columbia. They considered different steps in the process and provide an approximation of likelihood that the event occur in some order.

[37]Used Markov chain models in order to give the length of stay at the cardiac surgery department of Dutch Hospital.[38]Implemented Markov chain modeling for patient flow in which three of the cares were considered as severe, long stay,and rehabilitative care. The model was assigned in such a way that the cost of each and every care was assigned to model the expected cost for each and every group. This was beneficial to hospital managers. Besides, the probability values for each group can be adjusted and resulted information could be used for comparing various strategies.

[39]Discrete time Markov chain (DTMC) was applied to assess the re-admission probabilities of patients. Further, [40]applied (DTMC) to model to predict the number of inpatients, demonstrating how their model attained superior predictability compared to seasonal auto regressive integrated moving average model.

Patient flow can be seen as Markov chain because it involves movement of patients from one department to another. [35]Proposed a Markov chain to model the care process of doctor's consultation. Their study included 5 states *waiting, nurse care, examination, imaging ,checkouts*. In their study historical data were used to derive the transition probability among the states. The scope of this model is limited to doctor's consultation, it does not consider the entire patients received at the hospital. Their model do not provide a wider picture of the care process. In this report we use Markov chains to analyze patient flow through the entire hospital system.

CHAPTER THREE

METHODOLOGY

In this chapter the following is described: Data collection, Target population Model that was used.

3.1 Data collection

The basic data that was used for this report is secondary data, secondary data is the data that exist in recorded form such as patients' registers. This study we utilized the recorded data of the patients from hospitals record office, covering for a period of one year from January 2018 to December 2018.

3.2 Target population

A population is an entire group of people, objects or events a study wishes to investigate. The study population in this case was the total number of patients received at the hospital's registration office.

In this report we studied a population of 255,061 patients who were received for a period of one year from 2018 January to 2018 December at Kapsabet County Referral Hospital.

3.3 Model in use

To be able to understand typically the problem in clinical system, it is necessary to analyze patient flow volumes in the care system using a predictive model. Markov chain theory is the proposed model to be used describe the system. We used a Markov chain in order to model the process of change in patient flow population. Markov chains are a mathematical description of a system in which a transition from one state to another can be described with a certain probability. The transition probability does not depend on the previous states. Markov process is said to be memory less.

3.3.1 Markov chain theory

Markov chains, named after Russian mathematician Andery Markov is a type of stochastic process dealing with random process. There are two types of Markov chains. Discrete-time, a countable or finite process and countable-time an uncountable process. In this project we will deal with discrete-time Markov chains. These are stochastic process that satisfies three properties.

i. Discrete-time.

ii. Countable or finite states.

iii.Future location depends only on present state.

Definition

Markov chain is a stochastic process $\{x_k : k = 0, 1, 2, \text{âĂę}\}$ having the property that given the present state, the future is conditionally independent of the past.

A stochastic process $\{x_k : k = 0, 1, 2, \text{âĂę}\}$ with a state space of $S = \{1, 2, 3, \text{âĂę}\}$ is said to satisfy the Markov property if for every k and all states $\{i_1, i_2, \text{âĂę}, i_k\} \in S$

That is true that;

$P[x_k = i_k \mid x_{k-1} = i_{k-1}, x_{k-2} =, \text{âĂę}, x_1 = i_x] = P[x_k = i_k \mid x_{k-1} = i_{k-1}]$

The Markov property is satisfied if the future location of a particle depends on present location and not the past.

With this information we are ready to define discrete time Markov chain.

Definition

Discrete time Markov chain is a stochastic process that must satisfy the following restrictions.

i. Discrete time.

ii. Countable or finite state space.

iii.The future location depend on the present and not the past.

Now we need to describe a Markov chain as a probability since we are interested in the odd of moving from one state to another.

3.3.2 Transition probability

Transition probability is the probability of moving from one state say i to another state say j discrete number of n steps, denoted by P_{ij}^{n} .

We need to represent discrete time Markov chain with finite state space x_n . In order to represent state i and state j together, the most convenient way to do it is by using matrix P. By associating the i_{th} row and column of P with i_{th} state S and similarly with the state j transition probability matrix of the form P_{ij} will be achieved by arranging transition probability values P_{ij} in matrix form, forming Transition matrix.

$$P_{ij} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} \tag{3.1}$$

This is called transition matrix.

Each entry p_{ij} of p represent the probability that the patient movement transition from state i to j in one step.

Preposition

Every transition matrix has the following properties

All the entries are non-negative i.e

$p_{ij} \geq 0$

The sum of entries in each row is one i.e

$\Sigma_{i=1}^{n} p_{ij} = 1$

Proving the above preposition

$p_{ij} \geq 0$

Since probabilities can be represented as positive values.

For any $i = 1, 2,$ âĂę$, n$

$p_{i1} + p_{i2} + \cdots + p_{1n}$
$= P[x_k = 1 \mid x_{k-1} = i] + P[x_k = 2 \mid x_{k-1} = i] +$ âŃŕ $+ P[x_k = n \mid x_(n-1) = i]$
$= P[(x_k = 1) \cup (x_k = 2) \cup$ âĂę $\cup (x_k = n \mid x_{k-1} = i)]$
$= P[x_k \in s | x_{k-1} = i]$
$= 1$

Such a matrix with $\Sigma p_{ij} = 1$ is said to be a row stochastic.
Hence the transition matrix satisfy the preposition.

3.3.3 Estimation of transition matrices

In real data analysis, the transition matrix need to be estimated. First it is desirable to transform data to portray transitions per discrete time unit. When data is in desired form, it is reasonable to use maximum likelihood (ML) estimator.
$p_{ij} = \frac{N_{ij}}{\sum_k N_{jk}}$
So that for each possible transition,$i \to j$ to estimate the transition matrix. N_{ij} is defined as total number of observed transitions from state i to state j.

3.3.4 Markov states.

Patient flow was observed to be having a pattern of patients populations totals in the hospitals per day. This pattern was classified as high, medium, low and very low depending on the amount of the patients received in the hospital. In this report, the patient flow patterns were taken to be the Markov states. These states were are created depending on the levels of the flow.

These states are:
p_1 – High.
p_2 – medium.

p_3 – low.

p_4 – very low.

3.3.5 State classification

For a discrete-time Markov chains with the state space S and transition probabilities $P = (p_{i,j})$ $i, j \in S$, we say that there is a possible path from state i to state j if there is a sequence of state

$i = i_0 \to i_1 \to \cdots \to i_n = j$

Such that for all transitions along paths we have $p_{il-1}, i > 0, l = 1,$ âĂę$, n$. We will also use the phrase that the state j is accessible from state i. We say that two states communicate if there is a possible path from i to j and we use the notation $i \leftrightarrow j$ when the two states communicates.

If two states in Markov chain communicate, then they are said to be in same class. If the chain has only one class, it is said to be *irreducible*. The term irreducible refers to the fact that there is more than one class in Markov chain. We can think of a class as an individual state.

State i is said to be *recurrent* if and only if there is a probability of 1 that the process starting in state i will at some point return to state i.

State i is said to be transient if the probability that the process, stating in state$i$ will at some point return to state i is less than 1.

If state i is *recurrent* , then it is said to be positive recurrent if, starting in i, the expected time until the process returns to state i is finite. Also, positive recurrent, a periodic states are called *ergodic state*.

3.4 Regular transition matrix

A transition matrix is regular if some powers of that matrix contains all positive entries. A Markov chain is regular if its transition matrix is regular.

One of the many applications of Markov chains is to find long range predictions. It is not possible to make long range prediction with all transition matrices, but for a large set of transition matrices, long predictions are possible. Such predictions are always possible with regular transition matrices.

3.5 Homogeneity and Limiting distribution.

A Markov chain is homogeneous if its transition probabilities do not change over time. That is the probability of going from i to state j at time $t = 1$, is equal to the probability of going from state i to state j in some future period. Markov chain is *ergodic* if it is possible to go from any state to every state. Moreover, a Markov chain is called regular if some powers of transition matrix has only strictly positive elements.
Let the limiting distribution be v. We require the Markov kernel to be primitive. A Markov kernel is primitive if there exist an n such that $p_{ij}^n > 0$, for all values of i, j. If P has a primitive Markov kernel on finite space with limiting distribution v, then uniformly for all distribution of v.
$\lim_{n\to\infty} vP^n = V$
V i s also known as stationary distribution or a steady state.
$v = [v_1, v_2, \text{âĂę}, v_j]$ is a unique probability vector. The probability of starting at state $i = 1, 2, \text{âĂę}, j$ in the long run will settle at various values that are solutions of the equation $vp = V$
This represents matrix notation. The matrix notation for stationary distribution or a steady state is given by

$$\begin{bmatrix} v_1, & v_2, & \cdots & v_j \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix} = \begin{bmatrix} v_1, & v_2, & \cdots & v_j \end{bmatrix} \tag{3.2}$$

Expanding the above matrix, linear sets of equations formed are

$v_1 p_{11} + v_2 p_{21} + \cdots + v_1 p_{k1} = v_1$

$v_1p_{12} + v_2p_{22} + \cdots + v_jp_{k2} = v_2$

$v_1p_{1k} + v_2p_{2k} + \cdots + v_jp_{kk} = v_j$

Where

$v_1 + v_2 + \cdots + v_k = 1$

3.5.1 Measure for existence of limiting distribution

The Chapman-Kolmogorov equations enables one to compute the P_{ij}^{n+m} transition probabilities. These equations should be interpreted as computing the probability of starting in state i and ending in state j in exactly$n + m$ transitions through a path which takes into state k at the n_{th} transition. The equation is:
$P_{ij}^{n+m} = \Sigma P_{ij}^{n} P_{kj}^{m}$ for $n, m \geqslant$ all i, j
We use the Chapman-Kolmogorov equations in order to test the limiting probabilities of Markov chains. We compute the P^n transition probabilities for $i = j$ and $k = 0$. We assume that the time periods are indifferent.

3.6 Absorbing Markov chains

Definition.
Absorbing state; state i of a Markov chain is an absorbing state if $p_{ii} = 1$ Using the idea of absorbing state we can define absorbing Markov chain.
Definition.
Absorbing Markov chain; is an absorbing chain if and only if the following conditions are satisfied;
The chain has at least an absorbing state and
It is possible to go from any non-absorbing state to an absorbing state (perhaps in more

than one step).

Definition:

In an absorbing Markov chain, a state which is not absorbing is called transient

In order for Markov chain to end it must contain one of the absorbing state where the states cannot pass to another or other states.

An example

Suppose a Markov chain has a transition matrix

$$\begin{pmatrix} 0.3 & 0.6 & 0.1 \\ 0 & 1 & 0 \\ 0.6 & 0.2 & 0.2 \end{pmatrix} \tag{3.3}$$

The above matrix show that p_{12}, the probability of going from state 1 to 2, is 0.6 and that p_{22}, the probability of staying in state 2 is 1. Thus once state two is entered, there is no leaving. For this reason the state 2 is called an absorbing state.

Properties of absorbing Markov chains.

i. Regardless of the original state of an absorbing Markov chain; in a finite number of steps the chain will enter an absorbing and stay in that state.

ii. The powers of transition matrix get closer and closer to some particular matrix.

iii. Long term trend depend on initial state. Changing the initial state can change the final result.

It would be preferable to have a method for finding the final probabilities of entering an absorbing state without finding the powers of transition matrix.

3.6.1 Canonical form

This is the arrangement of Markov chains so that the absorbing states and transition matrix is separated.

Illustration

$$P = \begin{pmatrix} I & O \\ R & Q \end{pmatrix} \tag{3.4}$$

3.6.2 Fundamental matrix

Fundamental matrix for an absorbing Markov chain can be defined as a matrix M, where
$M = (I - Q)^{-1}$
Here
M – The fundamental matrix
I – The identity matrix
Q – Transition matrix
I and Q have the same size.

3.6.3 Probabilities of absorption.

The probability that the process will enter the j_{th} absorbing state if its state is in i_{th} transient state called probability of absorption.
It is given by B=MR
Where
M – Is the fundamental matrix.
R – Is the vector matrix from Markov chains.

3.7 Predictions in future time

Having the initial vector containing the current state of Markov chain process, then the future state of the process can be predicted by;
$P^n = p^0 p^n$
Where
p^0- is the initial vector.
p^n-is the transition matrix at n_{th} time.

3.8 Data processing and analysis

The collected data was captured in R software package version 3.5.0, where the processing and analyses was done.

CHAPTER FOUR

RESULTS AND DISCUSIONS

Data

The data that was used in this project, was the patient flow volumes received at Kapsabet county referral hospital from January 2018 to December 2019. The data of the patients included were the general medical patients. This is because this proportion make up the largest proportion of the patients received in this hospital an approximate of 59 percent of the total patients.

Data for all the months were included except for March and April 2018, as this is when the medical practitioners at this hospital were on strike and there were no patients observed during this period.

Markov chain analysis

The patient flow data was cleaned and used for analysis. The weekly mean (μ) on the data for patients at Kapsabet county referral hospital within a week was calculated. The deviations from the mean was also computed to construct the states of the patient flow using the standard deviation (σ). We classified the data according to the levels of flow population so that we had; high, medium, low and very low. The days of the week were classified according to the flow, having either high, medium, low or very low depending on the population of the patient flow in a day.

The pattern was either high, medium, low or very low depending on the numbers of the patients received in the hospital. At times, we could have a flow being so high, then it shoot down to either medium, low or very low. We used the flow pattern as the Markov states, where we had this patterns put to a matrix. We had different matrices for the days of the week.

The patient flow population was classified as either high, medium, low or very low using these categories;

We used the quartiles to come up with this kind of the limits of the states;

Very low $(k \geq \mu - \frac{\delta}{2})[0 - 152]$

Low $(\mu - \frac{\delta}{2}k \leq \mu + \frac{3}{4}\delta)[153 - 236]$

Medium $(\mu + \frac{3}{4}\delta \geq k \leq \mu + \frac{3}{4}\delta)[237 - 286]$

High $(k > \mu + \frac{3}{2}\delta)[2876 \geq \infty]$

Each day was classified as either high, low, medium or very low. we had state to state pairs e.g high-high, low-low, medium- medium etc. So in this case we ended up with sixteen pairs of these states.

The number of high days were recorded per week, divided by the total number of days in a week to have a probability of the flow being high. The same was done for the states; medium, low and very low to have the following transition matrices;

TRANSITION MATRICES

MON - MON TRANSITION MATRIX

$$p = \begin{pmatrix} 0.6120 & 0.3477 & 0.0327 & 0.0076 \\ 0.5326 & 0.3974 & 0.0549 & 0.0150 \\ 0.4210 & 0.4589 & 0.0920 & 0.0280 \\ 0.3908 & 0.4745 & 0.1021 & 0.0319 \end{pmatrix} \tag{4.1}$$

It was observed that this matrix is a regular one and has a higher probability 0.612 with the level high for this day. This probability is the highest registered probability on this day.

Its steady state matrix is reached when $p^8 = p^9 = p^{10} = p^n$ as shown in the appendix A.

$$p^n = \begin{pmatrix} 0.5710 & 0.3723 & 0.04437 & 0.01153 \\ 0.5709 & 0.3723 & 0.04437 & 0.01153 \\ 0.5709 & 0.3723 & 0.04437 & 0.01153 \\ 0.5705 & 0.3721 & 0.0443 & 0.011528 \end{pmatrix} \tag{4.2}$$

This steady state shows that probability of the flow being high in future on this day will be with probability 0.57, medium with probability 0.37, low with 0.04 and very low with 0.011.

TUE - TUE PROBABILITY MATRIX

$$p = \begin{pmatrix} 0.4075 & 0.4660 & 0.1096 & 0.0169 \\ 0.3652 & 0.4788 & 0.1313 & 0.0248 \\ 0.2805 & 0.4977 & 0.1787 & 0.0431 \\ 0.1903 & 0.5026 & 0.2385 & 0.0685 \end{pmatrix} \tag{4.3}$$

Its steady state matrix is reached at $p^5 = p^6 = p^n$, as shown in the appendix B. The steady state is given as bellow;

$$p^n = \begin{pmatrix} 0.3651 & 0.4774 & 0.1324 & 0.0254 \\ 0.3652 & 0.4775 & 0.1324 & 0.0254 \\ 0.3651 & 0.4774 & 0.1324 & 0.0254 \\ 0.3651 & 0.4774 & 0.1324 & 0.0254 \end{pmatrix} \tag{4.4}$$

It was observed that the flow in future will be with probabilities for high is 0.36, medium is 0.47, low is 0.13 and very low at 0.02 for this perpendicular day. It was observed that during Tuesdays the flow was lower than on Mondays where we had highest probability registered.

WEN - WEN TRANSITION MATRIX

$$p = \begin{pmatrix} 0.4100 & 0.4626 & 0.1034 & 0.0241 \\ 0.3651 & 0.4655 & 0.1293 & 0.0401 \\ 0.2992 & 0.4597 & 0.1683 & 0.0728 \\ 0.2678 & 0.4557 & 0.1870 & 0.0895 \end{pmatrix} \tag{4.5}$$

Its steady state transition matrix is reached when $p^6 = p^7 = p^n$, as we have in the appendix C

$$p^n = \begin{pmatrix} 0.3695 & 0.4635 & 0.1270 & 0.0403 \\ 0.3695 & 0.4635 & 0.1270 & 0.0403 \\ 0.3695 & 0.4634 & 0.1270 & 0.0403 \\ 0.3695 & 0.4634 & 0.1270 & 0.0403 \end{pmatrix} \tag{4.6}$$

It was observed that in future probability of the flow being high was 0.3695, medium is o.483, low is 0.127 and very low with 0.04 on this day.

THUR - THUR TRANSITION MATRIX

$$p = \begin{pmatrix} 0.3531 & 0.5223 & 0.1017 & 0.0229 \\ 0.3089 & 0.5319 & 0.1261 & 0.0331 \\ 0.2420 & 0.5389 & 0.1678 & 0.0513 \\ 0.1520 & 0.5353 & 0.2322 & 0.0806 \end{pmatrix} \tag{4.7}$$

Its steady state transition matrix is reached when $p^4 = P^6 = p^n$ as shown in the appendix D.

$$p^n = \begin{pmatrix} 0.308 & 0.529 & 0.127 & 0.033 \\ 0.308 & 0.529 & 0.127 & 0.033 \\ 0.308 & 0.529 & 0.127 & 0.033 \\ 0.308 & 0.530 & 0.127 & 0.033 \end{pmatrix} \tag{4.8}$$

It was observed that in future the probability of flow being on this day at high will be 0.30, medium at 0.529, low at 0.12 and very low at 0.033.

FRI - FRI TRANSITION MATRIX

$$p = \begin{pmatrix} 0.2625 & 0.5557 & 0.1443 & 0.0375 \\ 0.2284 & 0.5418 & 0.1784 & 0.0513 \\ 0.1837 & 0.5108 & 0.2317 & 0.0738 \\ 0.1459 & 0.4709 & 0.2858 & 0.0974 \end{pmatrix} \tag{4.9}$$

Its steady state is reached when the transition matrix is at $p^8 = p^9 = p^n$ as shown in the appendix E.

$$p^n = \begin{pmatrix} 0.222 & 0.534 & 0.186 & 0.054 \\ 0.222 & 0.534 & 0.186 & 0.054 \\ 0.222 & 0.534 & 0.186 & 0.054 \\ 0.222 & 0.534 & 0.186 & 0.054 \end{pmatrix} \tag{4.10}$$

It is evident that the future probability of flow on Fridays being high is 0.22, medium with 0.53, low at 0.186 and very low at 0.05

SAT - SAT TRANSITION MATRIX

$$p = \begin{pmatrix} 0.2101 & 0.5405 & 0.1794 & 0.0700 \\ 0.1837 & 0.5182 & 0.2012 & 0.0996 \\ 0.1539 & 0.4804 & 0.2281 & 0.1377 \\ 0.1282 & 0.4427 & 0.2521 & 0.1770 \end{pmatrix} \tag{4.11}$$

Its steady state matrix is reached when transition matrix $p^{12} = p^{13} = p^n$ as shown in the appendix F.

$$p^n = \begin{pmatrix} 0.179 & 0.515 & 0.212 & 0.1129 \\ 0.179 & 0.516 & 0.212 & 0.1132 \\ 0.179 & 0.515 & 0.212 & 0.1129 \\ 0.179 & 0.514 & 0.212 & 0.1128 \end{pmatrix} \tag{4.12}$$

It was observed that the future probability of flow on Saturday will be high with 0.17, medium at 0.51, low at 0.21 and at very low with 0.11.

SUN - SUN TRANSITION MATRIX

$$p = \begin{pmatrix} 0.1558 & 0.54543 & 0.2041 & 0.0949 \\ 0.1365 & 0.51760 & 0.2191 & 0.1268 \\ 0.1194 & 0.48660 & 0.2288 & 0.1652 \\ 0.0934 & 0.43490 & 0.2405 & 0.2313 \end{pmatrix} \tag{4.13}$$

Its steady state is reached when its transition matrix is at $p^6 = p^7 = p^n$ as shown in the appendix G

$$p = \begin{pmatrix} 0.1289 & 0.502 & 0.222 & 0.1466 \\ 0.1289 & 0.502 & 0.222 & 0.1466 \\ 0.1289 & 0.502 & 0.222 & 0.1466 \\ 0.1289 & 0.502 & 0.222 & 0.1466 \end{pmatrix} \tag{4.14}$$

It was observed that in future the probability of flow on Sunday will be high with 0.12, medium with 0.50, low with 0.22, and very low with 0.14.

4.1 Limiting distribution

The probability of going from i to state j at time $t = 1$, is equal to the probability of going from state i to state j in some future period. Markov chain is *ergodic* if it is possible to go from every state to every state(not necessarily in one move). Moreover, a Markov chain is called regular if some powers of transition matrix has only strictly positive elements. Let the limiting distribution be v. We require the Markov kernel to be primitive. A Markov kernel is primitive if there exist an n such that $p_{ij}^n > 0$, for all values of i, j. If P has a primitive Markov kernel on finite space with limiting distribution v, then uniformly for all distribution of v.

$\lim_{n \to \infty} vP^n = V$

V is also known as limiting distribution or steady state distribution or invariant distribution.

$v = [v_1, v_2, âĂę, v_j]$ is a unique probability vector. The probability of starting at state $i = 1, 2, âĂę, j$ in the long run will settle at various values that are solutions of the equation $vp = V$

The steady states for the flow for the different states were then plotted on a line graph to have a clear behavior of the states for high, medium, low and very low.

The figure below shows the weekly trends of steady state probabilities for a weekly patterns of patient flow.

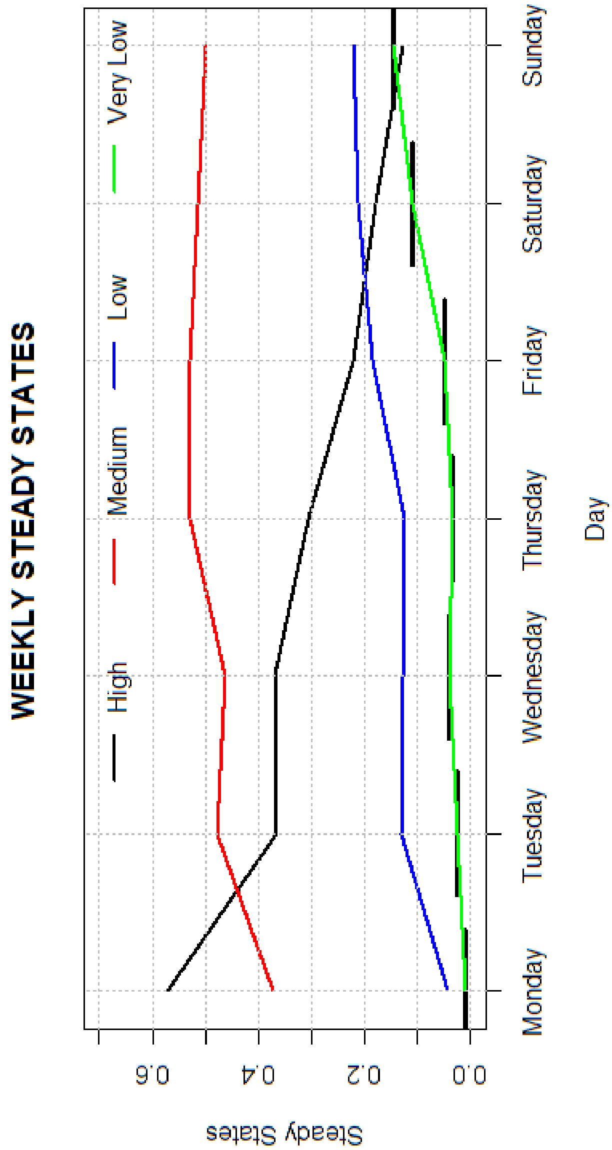
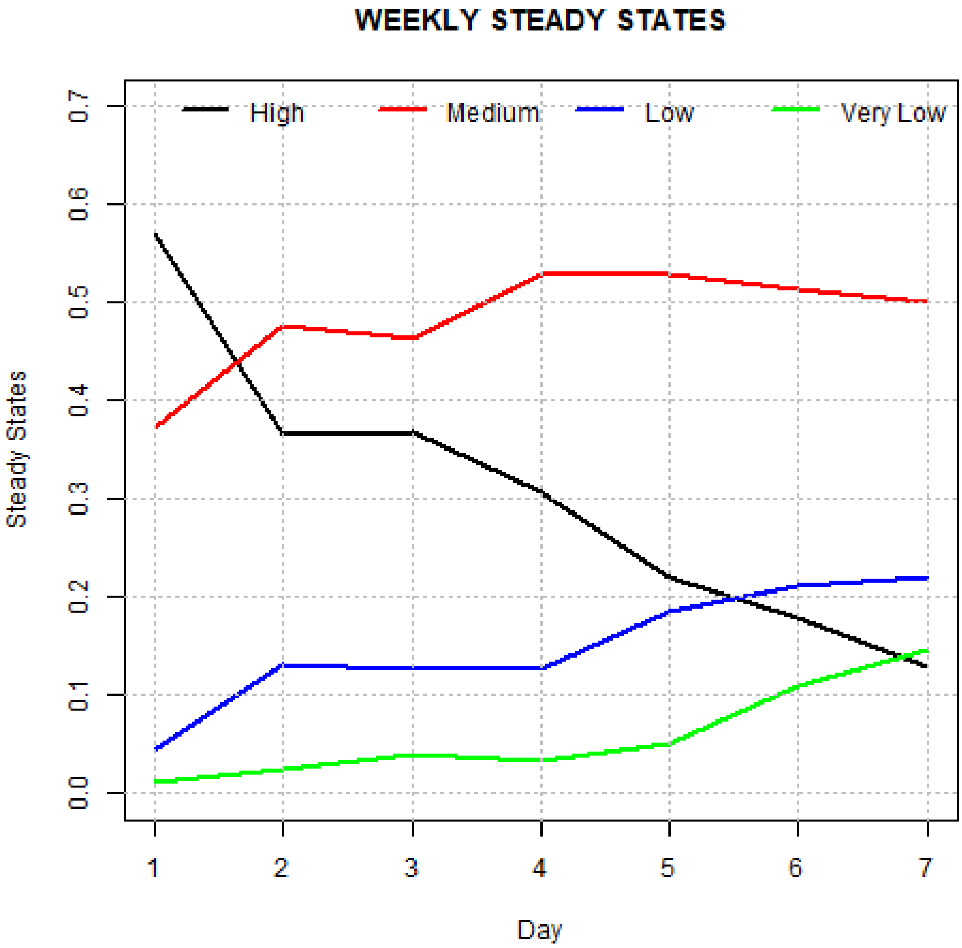

WEEKLY STEADY STATES
High
Medium
Low
Very Low
Monday
Tuesday
Wednesday
Thursday
Friday
Saturday
Sunday
Day
0.0
0.2
0.4
0.6
Steady States

Figure 4.1



WEEKLY STEADY STATES
High
Medium
Low
Very Low
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Steady States
1
2
3
4
5
6
7
Day

Figure 4.2

The above figure is comparing the weekly patterns of the patient flow using the steady states probabilities. It was observable that the flow will be high on Mondays, medium from Tuesdays to Thursdays, the flow will drop on Fridays to low over the weekend.

4.2 Model Validation

It is important to validate the transition matrices to account for any major deviation of steady states from the initial probabilities. We chose to do validation based on the shift off the steady state.
A statistical test is done to compare any steady states and their respective transition matrices at 95 percent confidence interval. This procedure follow all the states of transition; (high, medium,low and very low)
We used three days in a week to validate the model, these are Tuesday, Wednesday and Thursday

The procedure is as follow:
Test statistic
$z = \frac{(H_1 - H_2)}{\sqrt{\sigma_H}}$

Where
$\sigma_H = \frac{(1-H_1)H_1}{n} + \frac{(1-H_2)H_2}{n}$

H_1- Actual probability of a particular state.
H_2 - Steady state probability of a particular state.
σ_H - Standard error of the difference between the actual probability and a steady state of a particular state.
n - Sample size, and in this project we used a sample of 399 chosen arbitrarily.
Significance level,$\alpha = 0.05$
Decision rule: If the calculated value is between the critical values, the the null hypothesis is not rejected.

CALCULATIONS

Given that the sample size $n = 399$, is large number greater than 30, the central limit theorem is applied; $Z = \frac{(H_1 - H_2) - 0}{\sigma_H}$

TUE - TUE

Hypothesis

$$H_0 : H_1 = H_2, M_1 = M_2, L_1 = L_2, V_1 = V_2 H_a : H_1 \neq H_2, M_1 \neq M_2, L_1 \neq L_2, V_1 \neq V_2$$

Sample size $(n) = 399$ Significance level $\alpha = 0.05 percent.$

Decision rule; if the computed value of the test statistic Z is greater than or equal to ± 1.64 , then we reject the H_0.

$H_1 = 0.4075, H_2 = 0.365, M_1 = 0.4788, M_2 = 0.4775, L_1 = 0.1313, L_2 = 0.1324, V_1 = 0.0248, V_2 = 0.0254$

High

$$z = \frac{(0.4075 - 0.365)}{\sqrt{\frac{(1-0.4075)(0.4075)+(1-0.365)(0.365)}{399}}}$$

$$Z = 1.235$$

Medium

$$z = \frac{(0.4788 - 0.4775)}{\sqrt{\frac{(1-0.4788)(0.4788)+(1-0.4775)(0.4775)}{399}}}$$

$$Z = 0.0367$$

Low

$$z = \frac{(0.1313 - 0.1324)}{\sqrt{\frac{(1-0.1324)(0.0.1324)+(1-0.1324)(0.1324)}{399}}}$$

$$Z = -0.04592$$

Very low

$$z = \frac{(0.0248 - 0.0254)}{\sqrt{\frac{(1-0.248)(0.248)+(1-0.054)(0.054)}{399}}}$$

$$Z = 0.02137$$

The steady state and their initial matrices have their Z values between the range of ± 1.64. We therefore, do not reject the null hypothesis H_0 and conclude that, the two matrices are consistent with the data used.

WEN - WEN

Hypothesis

$$H_0 : H_1 = H_2, M_1 = M_2, L_1 = L_2, V_1 = V_2 H_a : H_1 \neq H_2, M_1 \neq M_2, L_1 \neq L_2, V_1 \neq V_2$$

Sample size $(n) = 399$

Significance level $\alpha = 0.05 percent$.

Decision rule; if the computed value of the test statistic Z is greater than or equal to ± 1.64, then we reject the H_0.

$H_1 = 0.41, H_2 = 0.3695, M_1 = 0.4655, M_2 = 0.4635, L_1 = 0.1293, L_2 = 0.1270, V_1 = 0.0401, V_2 = 0.0403$

High

$$z = \frac{(0.41 - 0.3695)}{\sqrt{\frac{(1-0.41)(0.41)+(1-0.3695)(0.3695)}{399}}}$$

$$Z = 1.17396$$

Medium

$$z = \frac{(0.4655 - 0.4635)}{\sqrt{\frac{(1-0.4655)(0.4655)+(1-0.4635)(0.4635)}{399}}}$$

$$Z = 0.00057$$

Low

$$z = \frac{(0.1293 - 0.1270)}{\sqrt{\frac{(1-0.1293)(0.1293)+(1-0.1270)(0.1293)}{399}}}$$

$$Z = 0.097$$

Very low

$$z = \frac{(0.0401 - 0.0403)}{\sqrt{\frac{(1-0.0401)(0.0401)+(1-0.0403)}{399}}}$$

$$Z = -0.0139$$

The steady state and their initial matrices have their Z values between the range of ± 1.64. We therefore, do not reject the null hypothesis H_0 and conclude that, the two matrices are consistent with the data used.

THUR - THUR

Hypothesis

$$H_0 : H_1 = H_2, M_1 = M_2, L_1 = L_2, V_1 = V_2 H_a : H_1 \neq H_2, M_1 \neq M_2, L_1 \neq L_2, V_1 \neq V_2$$

Sample size $(n) = 399$
Significance level $\alpha = 0.05$ percent.
Decision rule; if the computed value of the test statistic Z is greater than or equal to ± 1.64, then we reject the H_0.
$H_1 = 0.3531, H_2 = 0.0308, M_1 = 0.5319, M_2 = 0.529, L_1 = 0.1261, L_2 = 0.127, V_1 = 0.033, V_2 = 0.031$

High

$$z = \frac{(0.3531 - 0.308)}{\sqrt{\frac{(1-0.35531)(0.3531)+(1-0.308)(0.308)}{399}}}$$

$$Z = 1.3557$$

Medium

$$z = \frac{(0.5319 - 0.529)}{\sqrt{\frac{(1-0.5319)(0.5319)+(1-0.529)(0.529)}{399}}}$$

Z=0.08207

Low

$$z = \frac{(0.1261 - 0.127)}{\sqrt{\frac{(1-0.1261)(0.1261)+(1-0.127)(0.127)}{399}}}$$

z = -0.0383

Very low

$$z = \frac{(0.033 - 0.031)}{\sqrt{\frac{(1-0.033)(0.033)+(1-0.031)(0.031)}{399}}}$$

Z=0.01605

The steady state and their initial matrices have their Z values between the range of ± 1.64. We therefore, do not reject the null hypothesis H_0 and conclude that, the two matrices are consistent with the data used.

CHAPTER FIVE

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

Congestion and overcrowding of patients in public hospitals is becoming considerable hindrance to quality of care and so hospital managers and their staff have always been so interested in reducing congestion and overcrowding of patients in hospitals. In this project report developed an objective patient flow assessment through an analysis of Markov chains using weekly data from Kapsabet county referral hospital to check on the flow of patients. Transition matrices were computed for each day in a week to portray the weekly routine amount of the patient flow using the states; high, medium,low and very low. Steady state transition matrices were also computed for each day of the week to reflect the flow for each week. It was found that the patient flow had some pattern observed through the steady states. The probability of patient flow being high tends to be up on Mondays , medium on Tuesdays to Thursdays, on Fridays is when the steady states of being high starting to decrease upto Sunday. Through the analysis of patient flow using Markov chains, we have identified some pattern of how the patient flow so that we try to mitigate congestion problem in our hospitals. This project validates that the Markov chains analysis is a good model to analyze the patient flow.

5.2 Recommendations

This work can be useful to the hospital management, the staff and the ministry of health at the counties, as this enable them to plan their activities and resources accordingly to reduce overcrowding and match the resources with the flow.

Lastly, more research should be carried out at each referral hospital in different counties for planning.

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R SCRIPTS AND OUTPUTS

WEEKLY TRANSITION MATRICES AND THEIR LIMITING DISTRIBUTIONS

APPENDIX A

MON-MON TRANSITION MATRIX

```
>p=matrix(c(612,.3477,.0327,.0076,.5326,.3974,.0549,.015,.421,
.4589,.092,.028,.3908,.4745,.1021,.0319),nrow=4,ncol=4,byrow=TRUE)

> p
       [,1]   [,2]   [,3]   [,4]
[1,] 0.6120 0.3477 0.0327 0.0076
[2,] 0.5326 0.3974 0.0549 0.0150
[3,] 0.4210 0.4589 0.0920 0.0280
[4,] 0.3908 0.4745 0.1021 0.0319
```

MON- MON LIMITING DISTRIBUTION

```
> P^2
          [,1]      [,2]       [,3]       [,4]
[1,] 0.5764658 0.3695806 0.04288549 0.01102474
[2,] 0.5665813 0.3754229 0.04581558 0.01202446
[3,] 0.5517365 0.3842534 0.05028311 0.01355230
[4,] 0.5473389 0.3864377 0.05147940 0.01396399

> P^3
          [,1]      [,2]       [,3]       [,4]
[1,] 0.5719990 0.3722199 0.04421150 0.01147733
[2,] 0.5706855 0.3729238 0.04458066 0.01160378
[3,] 0.5687815 0.3740466 0.04514703 0.01179724
[4,] 0.5679181 0.3741299 0.04527524 0.01184322
```

```
> P^4
          [,1]      [,2]       [,3]       [,4]
[1,] 0.5714061 0.3725389 0.04437853 0.01153454
[2,] 0.5711820 0.3725913 0.04442110 0.01154949
[3,] 0.5709288 0.3727272 0.04449234 0.01157389
[4,] 0.5705167 0.3725408 0.04448517 0.01157363

> P^6
          [,1]      [,2]       [,3]       [,4]
[1,] 0.5713058 0.3725633 0.04439786 0.01154132
[2,] 0.5712203 0.3725328 0.04439886 0.01154207
[3,] 0.5711773 0.3725431 0.04440708 0.01154496
[4,] 0.5708226 0.3723223 0.04438269 0.01153882

>P^8
          [,1]      [,2]       [,3]       [,4]
[1,] 0.5712682 0.3725502 0.04439839 0.01154168
[2,] 0.5712004 0.3725092 0.04439410 0.01154063
[3,] 0.5711841 0.3725034 0.04439430 0.01154078
[4,] 0.5708368 0.3722783 0.04436771 0.01153389

P^n
          [,1]      [,2]       [,3]       [,4]
[1,] 0.5710109 0.3723841 0.04437890 0.01153665
[2,] 0.5709457 0.3723416 0.04437384 0.01153533
[3,] 0.5709333 0.3723335 0.04437287 0.01153508
[4,] 0.5705873 0.3721078 0.04434598 0.01152809

APPENDIX B
```

TUE - TUE TRANSITION MATRIX

```
> Q=matrix(c(.4075,.466,.1096,.0169,.3652,.4788,.1313,.0248,.2805,
.4977,.1787, .0431,.1903,.5026,.2385,.0685),nrow = 4,ncol=4,byrow=TRUE)

> Q
 [,1]   [,2]   [,3]   [,4]
[1,] 0.4075 0.4660 0.1096 0.0169
[2,] 0.3652 0.4788 0.1313 0.0248
[3,] 0.2805 0.4977 0.1787 0.0431
[4,] 0.1903 0.5026 0.2385 0.0685
```

TUE -- TUE LIMITING DISTRIBUTION

```
> Q^2
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3701983 0.4760577 0.1294640 0.02432496
[2,] 0.3652259 0.4772451 0.1322705 0.02540395
[3,] 0.3543911 0.4796128 0.1383038 0.02773773
[4,] 0.3410316 0.4824542 0.1458055 0.03065215

> Q^3
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3656558 0.4771088 0.1320168 0.02530874
[2,] 0.3650557 0.4772993 0.1323866 0.02544902
[3,] 0.3636417 0.4775597 0.1331448 0.02574454
[4,] 0.3618942 0.4778929 0.1340893 0.02611219
```

```
> Q^4
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3650918 0.4772602 0.1323478 0.02543545
[2,] 0.3650473 0.4773263 0.1324066 0.02545558
[3,] 0.3648351 0.4773179 0.1324918 0.02549106
[4,] 0.3645796 0.4773180 0.1326005 0.02553569

> Q^5
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3650343 0.4772983 0.1323952 0.02545262
[2,] 0.3650606 0.4773486 0.1324143 0.02545742
[3,] 0.3650017 0.4773059 0.1324137 0.02545973
[4,] 0.3649366 0.4772634 0.1324157 0.02546316

> Q^n
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3651719 0.4774948 0.1324551 0.02546532
[2,] 0.3652086 0.4775428 0.1324684 0.02546788
[3,] 0.3651721 0.4774951 0.1324552 0.02546533
[4,] 0.3651348 0.4774464 0.1324417 0.02546273
```

APPENDIX C

WEN-WEN TRANSITION MATRIX

```
> R=matrix(c(.41,.4626,.1034,.0241,.3651,.4655,.1293,.0401,.2992,
.4597,.1683,.0728,.2678,.4557,.187,.0895),nrow=4,ncol=4,byrow=TRUE)
```

```
> R
       [,1]   [,2]   [,3]   [,4]
[1,] 0.4100 0.4626 0.1034 0.0241
[2,] 0.3651 0.4655 0.1293 0.0401
[3,] 0.2992 0.4597 0.1683 0.0728
[4,] 0.2678 0.4557 0.1870 0.0895
```

WEN-WEN LIMITING DISTRIBUTION

```
> R^2
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3743865 0.4635217 0.1241171 0.03811573
[2,] 0.3690704 0.4632983 0.1272004 0.04046745
[3,] 0.3603597 0.4629427 0.1323150 0.04441253
[4,] 0.3560926 0.4627617 0.1348211 0.04635140

> R^3
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3700735 0.4633865 0.1266615 0.04005702
[2,] 0.3693646 0.4633123 0.1270416 0.04035488
[3,] 0.3682502 0.4632662 0.1276934 0.04085612
[4,] 0.3677036 0.4632436 0.1280132 0.04110200

> R^4
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3695369 0.4633827 0.1269893 0.04030663
[2,] 0.3694127 0.4633307 0.1270260 0.04034090
[3,] 0.3692682 0.4633218 0.1271083 0.04040451
[4,] 0.3691974 0.4633174 0.1271486 0.04043571
```

```
> R^5
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3694804 0.4633971 0.1270351 0.04033974
[2,] 0.3694307 0.4633480 0.1270282 0.04034041
[3,] 0.3694099 0.4633438 0.1270378 0.04034825
[4,] 0.3693997 0.4633418 0.1270425 0.04035210

> R^6
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3694852 0.4634139 0.1270451 0.04034527
[2,] 0.3694449 0.4633651 0.1270325 0.04034165
[3,] 0.3694398 0.4633615 0.1270329 0.04034238
[4,] 0.3694374 0.4633598 0.1270331 0.04034275

R^n
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3695930 0.4635508 0.1270834 0.04035778
[2,] 0.3695542 0.4635021 0.1270700 0.04035354
[3,] 0.3695514 0.4634985 0.1270691 0.04035323
[4,] 0.3695501 0.4634969 0.1270686 0.04035309
```

APPENDIX D

THUR -- THUR TRANSITION MATRIX

```
S=matrix(c(.3531,.5223,.1017,.0229,.3089,.5319,.1261,.0331,.242,
.5389,.1678,.053,.152,.5353,.2322,.0806),nrow=4,ncol=4,byrow=TRUE)

> S
```

```
       [,1]   [,2]   [,3]   [,4]
[1,] 0.3531 0.5223 0.1017 0.0229
[2,] 0.3089 0.5319 0.1261 0.0331
[3,] 0.2420 0.5389 0.1678 0.0513
[4,] 0.1520 0.5353 0.2322 0.0806
```

THUR -- THUR LIMITING DISTRIBUTION

```
> S^2
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3141103 0.5293000 0.1241549 0.03243707
[2,] 0.3089239 0.5299298 0.1273331 0.03381649
[3,] 0.3003216 0.5309258 0.1326354 0.03612231
[4,] 0.2874690 0.5323934 0.1406382 0.03960745

> S^3
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3093890 0.5298651 0.1270548 0.03369653
[2,] 0.3087311 0.5299424 0.1274604 0.03387283
[3,] 0.3076349 0.5300709 0.1281363 0.03416666
[4,] 0.3060164 0.5303169 0.1291663 0.03461236

> S^4
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3087898 0.5299368 0.1274250 0.03385740
[2,] 0.3087062 0.5299471 0.1274768 0.03387990
[3,] 0.3085671 0.5299645 0.1275632 0.03391741
[4,] 0.3083886 0.5300437 0.1277059 0.03397726

> S^6
```

```
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3087143 0.5299474 0.1274725 0.03387800
[2,] 0.3087040 0.5299493 0.1274793 0.03388090
[3,] 0.3086868 0.5299525 0.1274905 0.03388574
[4,] 0.3086919 0.5300104 0.1275202 0.03389643

> S^8
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3087056 0.5299504 0.1274790 0.03388073
[2,] 0.3087046 0.5299512 0.1274800 0.03388113
[3,] 0.3087030 0.5299525 0.1274816 0.03388181
[4,] 0.3087315 0.5300077 0.1274969 0.03388623

> S^n
          [,1]      [,2]      [,3]       [,4]
[1,] 0.3087146 0.5299685 0.1274842 0.03388226
[2,] 0.3087150 0.5299691 0.1274843 0.03388230
[3,] 0.3087156 0.5299702 0.1274846 0.03388237
[4,] 0.3087475 0.5300250 0.1274978 0.03388587
```

APPENDIX E

FRI-FRI TRANSITION MATRIX

```
>T =matrix(c(.2625,.5557,.1443,.0375,.2284,.5418,.1784,.0513,.1837,
.5108,.2317,.0738,.1459,.4709,.2858,.0974),nrow=4,ncol=4,byrow=TRUE)

> T
       [,1]   [,2]   [,3]   [,4]
[1,] 0.2625 0.5557 0.1443 0.0375
[2,] 0.2284 0.5418 0.1784 0.0513
```

```
[3,] 0.1837 0.5108 0.2317 0.0738
[4,] 0.1459 0.4709 0.2858 0.0974
```

FRI-FRI LIMITING DISTRIBUTION

```
> T^2
          [,1]      [,2]      [,3]       [,4]
[1,] 0.2278073 0.5383167 0.1811674 0.05265300
[2,] 0.2239589 0.5357530 0.1856121 0.05452188
[3,] 0.2182187 0.5319383 0.1924116 0.05738037
[4,] 0.2125644 0.5280626 0.1991187 0.06020722

> T^3
          [,1]      [,2]      [,3]       [,4]
[1,] 0.2237135 0.5355871 0.1859330 0.05465698
[2,] 0.2232069 .5352099 0.1864843 0.05489119
[3,] 0.2224949 0.5347725 0.1873678 0.05526046
[4,] 0.2217700 0.5342878 0.1882224 0.05561992

> T^4
          [,1]      [,2]      [,3]       [,4]
[1,] 0.2231832 0.5352112 0.1865322 0.05491032
[2,] 0.2230995 0.5351172 0.1865865 0.05493547
[3,] 0.2230289 0.5351098 0.1867160 0.05498750
[4,] 0.2229374 0.5350501 0.1868257 0.05503353

>T^6
          [,1]      [,2]      [,3]       [,4]
[1,] 0.2231052 0.5351383 0.1865999 0.05494005
```

```
[2,] 0.2230754 0.5350804 0.1865908 0.05493854
[3,] 0.2230866 0.5351278 0.1866242 0.05495014
[4,] 0.2230758 0.5351223 0.1866389 0.05495622

> T^8
          [,1]      [,2]      [,3]       [,4]
[1,] 0.2230849 0.5351040 0.1865998 0.05494128
[2,] 0.2230619 0.5350507 0.1865827 0.05493637
[3,] 0.2230835 0.5351051 0.1866038 0.05494281
[4,] 0.2230830 0.5351065 0.1866064 0.05494380
>
> T^n
          [,1]      [,2]      [,3]       [,4]
[1,] 0.2229762 0.5348455 0.1865114 0.05491542
[2,] 0.2229543 0.5347929 0.1864931 0.05491002
[3,] 0.2229774 0.5348483 0.1865124 0.05491571
[4,] 0.2229784 0.5348506 0.1865132 0.05491595
```

APPENDIX F

SAT - SAT TRANSITION MATRIX

```
> U=matrix(c(.2101,.5405,.1794,.07,.1837,.5182,.2012,.0996,.1539,
.4804,.2281,.1377,.1282,.4427,.2521,.177),nrow =4,ncol=4,byrow=TRUE)

> U
       [,1]   [,2]   [,3]   [,4]
[1,] 0.2101 0.5405 0.1794 0.0700
[2,] 0.1837 0.5182 0.2012 0.0996
[3,] 0.1539 0.4804 0.2281 0.1377
```

```
[4,] 0.1282 0.4427 0.2521 0.1770
```

SAT - SAT LIMITING DISTRIBUTION

```
> U^2
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1800155 0.5108189 0.2050087 0.1056342
[2,] 0.1775221 0.5085705 0.2082205 0.1098062
[3,] 0.1733416 0.5026653 0.2110099 0.1144031
[4,] 0.1697484 0.4981660 0.2141960 0.1191101

> U^4
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1767518 0.5072552 0.2084644 0.1104056
[2,] 0.1768441 0.5081322 0.2093491 0.1111878
[3,] 0.1758996 0.5061877 0.2092061 0.1115048
[4,] 0.1754119 0.5055284 0.2095696 0.1120770

> U^6
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1765550 0.5074169 0.2091530 0.1111426
[2,] 0.1769719 0.5086925 0.2097450 0.1114967
[3,] 0.1764349 0.5072460 0.2092316 0.1112733
[4,] 0.1763407 0.5070687 0.2092387 0.1113248

> U^8
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1767438 0.5080513 0.2094931 0.1113702
[2,] 0.1772022 0.5093789 0.2100488 0.1116706
[3,] 0.1767160 0.5079935 0.2094880 0.1113787
```

```
[4,] 0.1766714 0.5078769 0.2094501 0.1113645

> U^10
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1769815 0.5087463 0.2097896 0.1115337
[2,] 0.1774458 0.5100820 0.2103414 0.1118277
[3,] 0.1769654 0.5087027 0.2097739 0.1115268
[4,] 0.1769269 0.5085936 0.2097302 0.1115043

> U^12
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1772257 0.5094497 0.2100809 0.1116893
[2,] 0.1776913 0.5107881 0.2106329 0.1119829
[3,] 0.1772111 0.5094078 0.2100639 0.1116805
[4,] 0.1771733 0.5092996 0.2100194 0.1116569

>U^n
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1791986 0.5151210 0.2124197 0.1129329
[2,] 0.1796694 0.5164744 0.2129778 0.1132296
[3,] 0.1791839 0.5150789 0.2124024 0.1129236
[4,] 0.1791459 0.5149696 0.2123573 0.1128997
```

APPENDIX G

SUN-SUN TRANSITION MATRIX

```
> V = matrix(c(.1558,.54543,.2041,.0949,.1365,.5176,.2191,.1268,.1194,
.4866,.2288,.1652,.0934,.4349,.2405,.2313),nrow =4,ncol=4,byrow=TRUE)
```

```
>V
     [,1]    [,2]   [,3]   [,4]
[1,] 0.1558 0.54543 0.2041 0.0949
[2,] 0.1365 0.51760 0.2191 0.1268
[3,] 0.1194 0.48660 0.2288 0.1652
[4,] 0.0934 0.43490 0.2405 0.2313

> V^2
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1319580 0.5078796 0.2208240 0.1396136
[2,] 0.1299228 0.5041203 0.2218913 0.1441097
[3,] 0.1277718 0.5001681 0.2230636 0.1490405
[4,] 0.1242347 0.4936671 0.2250036 0.1572393

> V^3
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1292909 0.5030233 0.2223107 0.1456947
[2,] 0.1290081 0.5024421 0.2223971 0.1462411
[3,] 0.1287340 0.5019380 0.2225462 0.1468700
[4,] 0.1282929 0.5011535 0.2228156 0.1479269

> V^4
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1289580 0.5024230 0.2225049 0.1464780
[2,] 0.1288959 0.5022476 0.2224711 0.1465181
[3,] 0.1288610 0.5021833 0.2224900 0.1465983
[4,] 0.1288160 0.5021273 0.2225439 0.1467459
```

```
> V^5
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1289205 0.5023659 0.2225383 0.1465835
[2,] 0.1288866 0.5022422 0.2224891 0.1465591
[3,] 0.1288821 0.5022340 0.2224915 0.1465693
[4,] 0.1288877 0.5022709 0.2225179 0.1466010

>V^6
          [,1]      [,2]      [,3]      [,4]
[1,] 0.1289207 0.5023780 0.2225511 0.1466027
[2,] 0.1288904 0.5022609 0.2225000 0.1465700
[3,] 0.1288898 0.5022598 0.2225003 0.1465712
[4,] 0.1289019 0.5023086 0.2225232 0.1465882

>V^n
   [,1]      [,2]      [,3]      [,4]
[1,] 0.1289485 0.5024878 0.2226009 0.1466370
[2,] 0.1289187 0.5023716 0.2225494 0.1466031
[3,] 0.1289187 0.5023716 0.2225494 0.1466031
[4,] 0.1289316 0.5024221 0.2225718 0.1466178


> # LINE GRAPHS IN R ----
> # SETTING WORKING DIRECTORY
> # PREPARING DATA ----
> Data <-data.frame(High  = c(0.571,0.365,0.369,0.307,0.22,0.179,0.128),
+                   Medium = c(0.372,0.477,0.463,0.53,0.53,0.514,0.5),
+                   Low = c(0.044,0.132,0.127,0.126,0.186,0.212,0.22),
+                   Very Low = c(0.011,0.025,0.04,0.033,0.05,0.11,0.146),
```

```
+                   Days = 1:7)
> Data$Days <- factor(Data$Days,
+                     levels = 1:7,
+                     labels = c("Monday",
+                                "Tuesday",
+                                "Wednesday",
+                                "Thursday",
+                                "Friday",
+                                "Saturday",
+                                "Sunday"))
>
>
> # METHOD ONE (Days as Days)----
> plot(Data$Days,
+      Data$Very_Low,
+      type = "l",
+      ylim = c(0, 0.7),
+      xlim = c(1, 7),
+      xlab = "Day",
+      ylab = "Steady States",
+      main = "WEEKLY STEADY STATES")
> lines(Data$High, col = "black", lwd = 2)
> lines(Data$Medium, col = "red", lwd =2)
> lines(Data$Low, col = "blue", lwd = 2)
> lines(Data$Very_Low, col = "green", lwd = 2)
> abline(h=seq(from = 0,to=0.7,by=0.1),v=1:7,lty=3,col="grey")
>
> legend("topright",
+        legend = c("High","Medium","Low","Very Low"),
+        col = c("black", "red", "blue", "green"),
```

```
+        lwd = 2,
+        bty = "n",
+        horiz = TRUE,
+        trace = TRUE)
 xchar= 0.3196 ; (yextra,ychar)= 0 0.0622
 segments2(-3.485,-0.7635,1.958,4.679,0.6658,0.6658,0.6658,
 0.6658,dx=0.6392,dy=0)
>
> # METHOD TWO (Days as numbers)----
> plot(x = 1,
+      xlab = "Days",
+      ylab = "Steady States",
+      main = "WEEKLY STEADY STATES",
+      ylim = c(0, 0.7),
+      xlim = c(1, 7),
+      type = "n")
> lines(Data$High, col = "black", lwd = 2)
> lines(Data$Medium, col = "red", lwd =2)
> lines(Data$Low, col = "blue", lwd = 2)
> lines(Data$Very_Low, col = "green", lwd = 2)
> abline(h = seq(from=0,to=0.7,by=0.1),v=1:7,lty=3,col= "grey")
> legend("topright",
+        legend = c("High","Medium","Low","Very Low"),
+        col = c("black", "red", "blue", "green"),
+        lwd = 2,
+        bty = "n",
+        horiz = TRUE,
+        trace = TRUE)
  xchar= 0.3196 ; (yextra,ychar)= 0 0.0622
  segments2(-3.485 -0.7635 1.958 4.679 , 0.6658 0.6658 0.6658 0.6658,
```

dx=0.6392,dy=0,...)

STANDARD NORMAL DISTRIBUTION:

Table Values for AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158

2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997