MODELING THE EFFECTS OF INTERFERENCE IN FERTILITY RATE: A CASE STUDY OF RWANDA, INDONESIA AND KENYA

BY

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Declaration

This thesis is my own work and has not been presented for a degree award in any other institution.

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Dedication

To My late dad James Odero, My mum Elsha, My son Reinhard and daughter Rehema

Abstract

Many studies have been done on fertility for many years. However, very little has been documented in the existing literature concerning modeling of fertility in the presence of interference, yet interference to fertility is a common phenomenon. In this study fertility data sets for Rwanda, Indonesia and Kenya were modeled before and after interference. The parameters of the model were estimated by the maximum likelihood estimation method. The model life table approach was used to determine the Net Fertility Value, F_0. A relationship between fertility rate in the presence of interference and population growth was also determined. Using Akaike's Information Criteria, (AIC), it was established that amongst the distributions fitted; Gamma, Weibull and Lognormal, Gamma gave the best fit for the fertility rate data, for all the countries studied, and interference simply shifts the Gamma distribution parameters. The result of this study would help the Governments to understand fully the effect of interference on fertility rate and plan for it. Demographers would also benefit from this study since it can be used to project population growth after an interference.

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LIST OF ABBREVIATIONS AND ACRONYMS

ASFR : Age Specific Fertility Rate

CBR : Crude Birth Rate

CWR : Child Woman Ratio

DHS : Demographic and Health Survey

IDHS: Indonesia Demographic and Health Survey

GFR: General Fertility Rate

GRR: Gross Reproductive Rate

KDHS: Kenya Demographic and Health Survey

NFR: Net Fertility Rate

NRR: Net Reproductive Rate

RDHS: Rwanda Demographic and Health Survey

TFR : Total Fertility Rate

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CHAPTER 1

INTRODUCTION

1.1 Background Information

The term fertility in demography context, refers to the actual production of children and not the physical capability to produce them, which is termed as fecundity. Demographers have always measured how quickly a population is growing, by determining how frequently people are added to the population by being born. This has been done by measuring fertility rate of a population which can be done in two broad ways namely; the period measures and the cohort measures. The period measures are measures which are based on a cross section of a population in one year. They include the Crude Birth Rate, CBR (the number of live births per 1000 women of a population in a given year), the General Fertility Rate, GFR (the number of live births per 1000 women between ages 15-49 years in a given year) and the Child Woman Ratio, CWR (the number of children under 5 years of age per 1000 women aged 15-49 years in a given year). On the other hand, cohort measures are measures which follow the same people over a period of decades. They include:

1. Age Specific Fertility Rate (ASFR), which refers to the annual number of live births per woman in a particular age group expressed per 1000 women in that age group;

$$ {}_nf_x = \frac{{}_nB_x}{{}_nF_x} \times 1000 \tag{1.1} $$

where,

${}_nf_x$ = Age Specific Fertility rate for age group x to $x + n$

${}_nB_x$ = Number of births occurring to women in the age group x to $x + n$

${}_nF_x$ = Number of females in the age group x to $x + n$

n = number of years in the age interval (normally 5 years)

2. Total Fertility Rate (TFR), which refers to the average number of children a woman would potentially have, were she to fast forward through all her child bearing years in a

given year, under all the age specific fertility rates for that given year. TFR represent the sum of annual age specific fertility rates computed for each age group in the childbearing period.

$$TFR = \left(n \left[\sum_{i=0}^{6} {}_{n}f_{x+in} + {}_{4}\, f_{45} \right] \right) \times 1000 \tag{1.2}$$

Barret, Bogue and Anderson [3] describe total fertility rate as a synthetic rate which is neither based on fertility of any real group of women, because this would involve waiting until they complete childbearing, nor based on counting up of the total number of children actually born over her reproductive lifetime, but based on the age specific fertility rate of women in their child bearing years, which in conventional international statistical usage is ages 15 - 49 years. TFR is therefore a measure of fertility of an imaginary woman who passes through her reproductive life and is subjected to all the age specific fertility rates for ages 15-49 years that were recorded for a given population in a given year. This measure represents the number of children that would be born to a hypothetical cohort of 1,000 women who follow a set of a current schedule of age specific fertility rates, assuming that none of the women die before reaching the end of the childbearing period.
According to Onoja and Osayomore [26], TFR is not only a more direct measure of the level of fertility of a population but also, an indicator of the potentiality for population change in a country. A rate of two children per woman is considered to be the replacement rate for a population, leading to stability in terms of total numbers, a rate of above two children would mean that a population is growing in size, while a rate of below two children would mean that a population is declining in size and growing older.

1.2 Fertility In Response to Interference

In this context, the term interference, refers to a situation of large scale strike of unanticipated phenomenon such as high magnitude earthquake, major floods, Wars and Genocides, which leave many people dead and thousands others displaced. Many investigators like Preston [30], Montogomery and Cohen [24], Guarcello [44, 13] and Palloni and Rafalimanana [28] have long observed that a strike of an interference in a population may cause big losses of assets, lives and displacements. Households may then have an incentive to increase the number of children ever born, and as a result, a positive fertility response in excess of replacement effects may be experienced.

1.3 Basic concepts and Definitions

1.3.1 Baby boom and Baby bust

Baby boom refers to a dramatic increase in fertility rates normally after a strike of an interference. For example, there was increase in fertility rates in the United states, Canada and New Zealand during the period following the world war II (1947-1961). On the other hand, baby bust refers to a rapid decline in fertility rates. Baby bust period normally follow immediately after baby boom period.

1.3.2 Demography

This is the scientific study of human populations primarily with respect to their size, their structure and their development. Demographers seek to understand population dynamics by investigating three main demographic processes: birth, migration and aging (including death) all which contribute to changes in populations.

1.3.3 Epidemiology

The science concerned with the study of the factors determining and influencing the frequency and distribution of disease, injury and other health related events and their causes in a defined human population.

1.3.4 Fertility and Fecundity

Demographers refer to fertility as the product or output of reproduction (actual live births), and Fecundity as the physiological ability to give birth which is manifested roughly in the period between menarche and menopause in women. Biologists do the opposite by referring to fertility as the capacity to produce a baby and fecundity as the realization of actual production hence should not be a source of confusion in this context. In demography, fertility is concerned with the number of live babies that women give birth to even if they have subsequently died. A fertile woman therefore, is one who has borne live children. Hence, a woman is not fertile until she has actually given birth to a live child. Prior to that, she will be considered a fecund woman if she is within a certain age range and there is no physical reason why she should not have children.

1.3.5 Fertility rate

This refers to live births per 1000 women, categorized according to a specific composition of mothers in a population. Examples are such as CBR, GFR,CWR, ASFR and TFR among others.

1.3.6 Model Life table

A life table is a mathematical model that portrays the mortality conditions at a particular time among a population and provides a basis for estimating longevity. A model life table was first developed in the 1950,s by the United Nations to study mortality conditions of populations. Although life tables were developed for the study of mortality UN,[48], they can be applied to any other 'failure' process so long as the process is measured in time (varies with age or some other measure of duration). Shepad and Greene,[46] in 2003, define Model life table as a table of data on survivorship and age specific reproductive rates of individuals within a population. A life table is usually based on a cohort of some arbitrary number of births (often 100,000). This number is denoted as l_0 and is referred to as the radix of the life table.
Life tables are used by demographers, public health workers, actuaries, and many others in studies of mortality, longevity, fertility, population growth, as well as in making projections of population, and in many other areas.
Our study applies the model cohort life table by Coale and Demeny, [6] which conceptually traces a cohort of newborn babies through their entire life under the assumption that they are subjected to the current observed schedule of age-specific death rates.

1.3.7 Modeling and Demography

Ader [1] states that Modeling is a form of scientific approach, often used to express the reality and its dimensions in precise terms. A model is therefore a simplified and mainly a mathematical representation of reality. Modeling of demographical processes as defined by Clogg and Eliason [5] is an attempt to represent demographic processes in the form of mathematical function or set of functions relating two or more measurable demographic variables. Since demographic models attempt to represent reality, they are based to a greater extent on actual data which are of course random in nature hence are statistical in nature. McCullagh [23], states that statistical Models include issues such as statistical characterization of numerical data and estimation of the probabilistic future behavior of a system based on past behavior. Statistical models therefore rely on probabilistic forms of description that have wide application over all areas of science. In a statistical model, randomness is present, and variable states are not described by unique values, but rather by probability distributions while on the other hand, mathematical (deterministic) models are described by unique values and grow out of equations that determine how a system changes from one state to the next (differential equations) or how one variable depends on the value or state of other variables (state equations).

1.3.8 Parametric and non Parametric Modeling

Parametric modeling covers techniques that rely on data belonging to a particular distribution and assumes that the structure of the model is fixed. Parametric models assume some finite set of parameters θ.

Given the parameters, future predictions, x are independent of the observed data, D.

On the other hand, non parametric modeling covers techniques that do not rely on the assumptions that the data are drawn from any particular distribution. The structure of the model is not specified in advance but is instead determined from the data. Non-parametric does not mean that the model does not include parameters, but that the number of parameters is not fixed in advance to specify a statistical model. Non parametric models assume that the data distribution cannot be defined in terms of a finite set of parameters. But, they can often be defined by assuming an infinite dimensional parameter θ and usually, θ is thought of as a function.

1.3.9 Stable Population and Stationary Population

A stable population is one that has had constant birth rates and death rates for such a long period of time that the number of people in every age group remains constant. A stable population does not necessarily remain fixed in size. It can be expanding or shrinking. Therefore, a population is said to be stable when both its growth rate and its relative age distribution do not change over time.
A stationary population refers to a population whose total number and distribution by age do not change with time hence unchanging in size (The difference in birth rate and death rate is zero). Assuming no migration, such a hypothetical population can be obtained if the number of births per year remained constant (usually assumed at 100,000) for a long period of time and each cohort of births experienced the current observed mortality rates throughout life. The annual number of deaths would thus equal 100,000 also and there would be no change in size of the population.

1.4 Statement of the problem

Interference continues to affect fertility globally. The World War I in the years 1914-1918, the World War II in the years 1939-1945, the Genocide in 1994 in Rwanda, the Tsunami in 2004 in Indonesia and the Post election Violence in 2008 in Kenya are some of the interferences that left thousands of people dead and many others displaced. Using the Rwanda, Indonesia and Kenya Demographic and Health Survey (DHS) data sets, we model fertility rates for the three countries before and after interference with the aim of determining the effect of interference on the fertility rates.

1.5 Objectives of the study

1.5.1 General Objective

To model human fertility rate data sets of Rwanda, Indonesia and Kenya in the presence of interference

1.5.2 Specific Objectives

1. To fit a model to interference free data sets of Rwanda, Indonesia and Kenya.
2. To determine the parameter estimates of the model fitted to interference free data sets of Rwanda, Indonesia and Kenya.
3. To fit a model to data affected by interference of Rwanda, Indonesia and Kenya.
4. To determine the parameter estimates of the model fitted to data affected by interference of Rwanda, Indonesia and Kenya.
5. To determine the effect of interference on fertility rate data sets of Rwanda, Indonesia and Kenya.

1.6 Significance of the study

A strike of interference in a country lead to an increase in the fertility rate of a country, which in turn lead to an increase in the demand for antenatal and postnatal care. Maternal clinics may then need to be increased to cater for the rise in the fertility rates. The results of this study are geared towards helping Governments to understand fully the effect of interference and plan for it. Demographers would also benefit from this study since it can be used to project population growth after an interference.

1.7 Justification of the study

This study is relevant in that it fills the gap left by researchers as little effort has been put in incorporating the effect of interference in modeling fertility rate.

1.8 Methodology

1.8.1 Some models of fertility rate

A variety of models, both parametric and non parametric have been proposed in literature to describe age specific fertility patterns, and several of them have been found to provide good fits to both human and non human fertility data. However, not much work has been done in incorporating the effect of interference in modeling of fertility data. The following are some of the models which have been used in literature for fitting fertility curves: the Hadwiger model [14], the modified Gamma model which was proposed by Hoem and Rennermalm [16], the Brass polynomial [4], the cubic spline which was proposed by Hoem and Rennermalm [16] in 1978 and the quadratic splines [43].
The Hadwiger function [14] is given by,

$$f(x) = \frac{ab}{c}(\frac{c}{x})^{\frac{3}{2}}exp[-b^2(\frac{c}{x} + \frac{x}{c} - 2)] \tag{1.3}$$

where,
$f(x)$ is the fertility rate at age x of the mother, the parameter a is related to the total level of fertility, the parameter b determines the height of the curve and the parameter c is related to the mean age of motherhood.

The Gamma distribution modified for fertility analysis [16] is given by,

$$f(x) = R\frac{1}{\Gamma(b)c^b}(x-d)^{b-1}exp[-\frac{(x-d)}{c}] \quad for \quad x > d \tag{1.4}$$

where,

$f(x)$ is the fertility rate at age x of the mother, d represents the minimum age at child-bearing, parameter R determines the level of fertility. The parameters b and c have no direct demographic interpretation, but are related to the mode, m, the mean, μ and the variance, σ^2, of the function such that; $c = \mu - m$ and $b = \frac{\sigma^2}{c^2}$.

The Brass polynomial [4] is given by,

$$M_x = c(x-d)(d+w-x)^2 \tag{1.5}$$

where,

M_x is the fertility rate at age x of the mother, c is a measure of level of fertility but cannot be interpreted as TFR. Parameter d is the lower age at fertility and w is the length of reproductive period.

The cubic spline , which is a piecewise cubic [16] splines is given by;

$$f(x) = a + b(x-m) + c(x-m)^2 + \Sigma_{j=1}^{n} d_j(x-m-k_j)^3 D_j \tag{1.6}$$

where,

where $D_j = 0$ if $x - m \leq k_j$ and $D_j = 1$ if $x - m > k_j$, m is the minimum child bearing age, $x \geq m$, k_j are the knots, n is the number of knots, and a, b, c and d_j are the coefficients that are estimated.

The quadratic spline [43] is a piecewise quadratic function which is given by;

$$f(x) = a + b(x - m) + \Sigma_{j=1}^{n} c_j (x - m - k_j)^2 D_j \tag{1.7}$$

where,

where $D_j = 0$ if $x - m \leq k_j$ and $D_j = 1$ if $x - m > k_j$, m is the minimum age, $x \geq m$, k_j are the knots, n is the number of knots, and a, b and c_j are the coefficients that are estimated. In our study, we modeled fertility rate data both before and after interference with a major aim of determining the effect of interference on fertility rate. We focused on both interference free data sets (Rwanda 1992, Indonesia 2002 and Kenya 2003 data sets) and also the data sets which had interference effect in them (Rwanda 2000, Indonesia 2007 and Kenya 2009 data sets). Using the model life table approach, we also determined Net Fertility Value, F_0 and related it to population growth.

1.8.2 Fitting model to data

Fitting model may involve a process of selection of the best fitting distribution function from a predefined family of distributions. This practice requires judgment and expertise and generally follows an iterative process of model choice, parameters estimation, and quality of fit evaluation. In R software environment, which was developed by the R core team [41], the package 'fitdistrplus' provides functions for fitting distributions to different types of data sets (continuous, censored or non censored data and discrete data). The package also allows for different estimation methods (maximum likelihood, moment matching, and maximum goodness of fit estimation).

Method of moment matching

The method of moments involves constructing estimators of the parameters basing on matching the sample moments with the corresponding distribution moments.

Method of maximum likelihood

The likelihood of a set of data refers to the probability of obtaining that particular set of data given the chosen probability model. Maximum likelihood thus begins with the mathematical expression called likelihood function of the sample data, the expression which contains the unknown parameters. The values of the unknown parameter that maximizes

the sample likelihood are the maximum likelihood estimates (MLE).
In R environment, we get the MLE by either of the following three statements:
'mle' included in the 'stats4' package in the R software, 'fitdistr' included in the 'MASS' package of R software or the 'fitdist' included in the 'fitdistrplus' package of R software. The statement 'mle' allows to estimate parameters for every kind of probability density function , it needs only to know the likelihood analytical expression to be optimized. However, arbitrary values of parameter estimates need to be supplied as starting values (estimates got by the method of moments can be used at this stage).
In both the 'MASS' and the 'fitdistrplus' packages are available 'fitdistr' and 'fitdist' functions respectively for maximum likelihood fitting of the univariate distributions without any information about likelihood analytical expression. It is enough to specify a data vector, the type of pdf and the list of starting values for iterative procedure.
In our study we used the maximum likelihood estimation method to estimate the parameters of the Gamma, Weibull and Lognormal distribution functions.
The Gamma distribution is defined by;

$$f(x) = \frac{1}{\Gamma\alpha\beta^{\alpha}} x^{\alpha-1} e^{\frac{-x}{\beta}} \quad \alpha > 0, \quad \beta > 0$$

where; α is the shape parameter , $\frac{1}{\beta}$ is the rate parameter, β being the scale parameter.
The Weibull distribution is defined by;

$$g(x) = \alpha\beta(\beta x)^{\alpha-1} e^{-(\beta x)^{\alpha}}, \quad for \quad x > 0, \quad \alpha > 0, \quad \beta > 0$$

where, α is the shape parameter , β is the scale parameter.
The Lognormal distribution is defined by;

$$q(x) = \frac{1}{\sqrt{2\pi\sigma^2}x} exp[-\frac{(ln(x)-\mu)^2}{2\sigma^2}], \quad for \quad x > 0, \quad \sigma > 0, \quad -\infty < \mu < \infty$$

Where μ is the shape parameter (the mean of the random variables logarithm), σ is the scale parameter (the standard deviation of the random variables logarithm). Akaike's information Criteria (AIC) was also used to estimate the quality of fit of each model. AIC is a measure of the relative quality of statistical models for a given set of data. Given a collection of models for the data, AIC estimates the quality of each model, relative to each of the other models. AIC is founded on information theory and it offers a relative estimate of the information lost when a given model is used to represent the process that generates the data hence from among the candidate models, the model that minimizes the information loss (the model with the lowest AIC value), is selected to be the best fitting model to the set of data.

1.8.3 Source of data

Our data sets were obtained from the Demographic and Health Survey Program (DHS). A body which since 1984, collects and disseminates nationally representative data on health and population on developing countries. The project is implemented by ICF international and funded by the United States Agency for International Development (USAID) and the access of the data sets has been made open to researchers since September 2013. We obtained and used the following data sets from the above named body.

- Rwanda 1992, Rwanda 2000 and Rwanda 2005 Demographic and Health survey data sets
- Indonesia 1997, Indonesia 2002, Indonesia 2007 and the Indonesia 2012 Demographic and Health survey data sets
- Kenya 2003, Kenya 2009 and Kenya 2014 Demographic and Health survey data sets

CHAPTER 2

LITERATURE REVIW

Much has been documented in literature on the effect of interference on fertility. Stein and Susser [45] in 1975, investigated the effect of massive famines on fertility in Neitherland, China and Bangladesh and observed that fertility had reduced during the famine interference and later on rose up sometime after the interference. In 1999, Lindstrom and Berhanu [21] analysed the impacts of conflict on fertility in Ethiopia and documented a sharp temporary decline in fertility during the early years of the violence, which was followed by a high increase in fertility thereafter. Stiegler [47] in 2006 , analysed fertility before and after the 1994 genocide in Rwanda and documented a decreasing trend of TFR before the Genocide and a sharp rise after the Genocide. A paper by the Nairobi chronicle research group [25] in 2008 also reported that the Kenya population had increased rapidly in the months which followed the post election violence. Hosseini and Abbasi [18] in 2013 investigated the impact of the 2004 Bam erthquake in Iran and documented that Iran's fertility had declined in the year 2004 and then rose in the years 2005-2007.

Modeling fertility curves has also attracted the interest of demographers for many years, and a variety of mathematical models have been proposed in order to describe the age specific fertility pattern of populations. The Hadwiger function proposed by Hadwiger in 1940 [14] was one of the earliest models which was fitted to age specific fertility data. But, this model had a problem of overestimating fertility at the oldest fecund ages. Other models have been proposed by various researchers, for fitting age specific fertility curves. For instance, the Brass polynomial [4] proposed in 1960, the modified Gamma and modified Beta distribution functions [16] both proposed in 1978, the cubic splines [16] proposed in 1978 and also the quadratic splines Schmertmann [43] proposed in 2003. However, the polynomial and spline models only fit fertility curves when elevated to a suitable degree. In 1981, Hoem *et.al* [17], compared the variations in fits of the cubic splines, the Gamma, the Hadwiger and the Brass functions in smoothing human fertility, using contemporary Denish fertility data and documented that among the models, the cubic spline fitted best.

The Gamma and the Hadwiger functions were second and still fit the data well, while the Beta and the Brass functions were less accurate.

Schmertmann [43] in 2003 fitted a quadratic spline to the age pattern of fertility, but this model required 13 parameters to estimate making it a bit complex. In the year 2000, Gage, [11] extended the application of the Gamma distribution function, the Hadwiger function and the Brass polynomial to several non-human mammalian populations (primates , Asian elephants, and PrzewalskiŠs horse (an extinct species)) . He tested all the three models and documented that Gamma model provided the best fit for fecundity of the non human populations. Otumba in 2012 [27], developed a model for optimal fish harvesting using Leslie Matrix. The fish species fecundity data was observed to follow a Gamma distribution. Jenna *et.al* [19] in 2015 used a multi level longitudinal data to investigate the fertility response to unanticipated mortality shock that had resulted from the 2004 Tsunami. They observed a positive association between exposure to the 2004Tsunami and subsequent fertility.

For all the models described above, modeling fertility rate with emphasis on interference effect is scarce in literature. Our study models the fertility rates of the data sets of Rwanda, Indonesia and Kenya both in the presence and in the absence of interference with an aim of determining the effect of interference on the fertility rate.

CHAPTER 3

MODELING NET FERTILITY VALUE IN RELATION TO POPULATION GROWTH

3.1 Introduction

Population variables depend on properties of the individuals that compose the population. The two basic parameters of a population are the individuals likelihood of surviving and the individuals likelihood to produce offsprings which both depend on the individual's age. The main parameter estimated by demographic analysis to describe the potential for population growth is the Net Fertility Rate (NFR). In our study , we used the model life table approach adapted from Coale and Demeny [6] to estimate the Net fertility value F_0 (commonly known as the Net Fertility Rate (NFR)), which we then linked to population growth.

3.2 Model life table Assumptions

It is assumed that in a life table;

- The population is closed . This is a population whose net migration is Zero hence not affecting the size of the population. Change in size of a closed population is as a result of number of births and number of deaths only.
- The population is stationary. This refers to a population in which none of the population variables change overtime. The annual number of births, the annual number of deaths, population size and the sizes of age groups are all constant.
- Deaths and births are evenly spread over the year.

3.3 Some Basic Model Life Table Variable Notations

Table 3.1: Some Basic Model Life table Variables

Variable	Definition
l_0	The radix of the life table. This is a cohort of some arbitrary number of births on which the life table is based.
l_x	The number of individuals who survive to age x
s_x	The probability that an individual survives to age x. $s_x = \frac{l_x}{l_0}$
p_x	The probability that an individual, alive at age x, survives to age $(x + 1)$. $p_x = \frac{l_{x+1}}{l_x}$
d_x	The number of deaths between ages x and $(x + 1)$ $d_x = l_x - l_{x+1}$
q_x	The probability of an individual alive at age x, dying in the age intervals x to $x + 1$ $q_x = \frac{d_x}{l_x}$
m_x	The number of female offsprings produced per individual at age x

3.3.1 Basic Reproduction Number

In epidemiology, the transmissibility of an infection according to Fraser [10], can be quantified by its basic reproduction number R_o, which is defined as the mean number of secondary infections produced by a single infection into a completely susceptible host population. For many simple epidemic processes, this parameter determines a threshold: whenever $R_o > 1$, a typical infective gives rise, on average, to more than one secondary infection, leading to an epidemic. In contrast, when $R_o < 1$, infectious individual typically give rise, on average, to less than one secondary infection, and the prevalence of infection cannot increase.

Lawi [20], in his model of Malaria-meningitis co-infection defined the basic reproduction number R_o, as the number of secondary (or meningitis) infections due to a single malaria (or a single meningitis infective) individual. When $R_o < 1$, then an infectious individual causes on average less than one new infection and the disease does not invade the population, while on the other hand when $R_o > 1$, then an infectious individual causes on average more than one new infection and the disease invades and persists in the population.

In our context, Basic reproduction number R_0, may represent the Net Reproduction value (number) or the Net Reproduction Rate (NRR), which refers to the average number of female offsprings produced by an individual in her reproductive lifetime into a population. The word 'individual' is used here to refer to a woman of child bearing age which in conventional statistics usage is a woman aging between 15 to 49 years old.

3.3.2 Determination of Net Reproduction Number and Net Fertility Number

Net reproduction number, R_0 refers to the average number of female offsprings produced by an individual in her reproductive lifetime into a population. The word 'individual' in this context, refers to a woman of child bearing age (15-49 years). Using the model life table [6];

s_x = probability that an individual survives to age x

m_x = the number of female offsprings produced
by an individual at age x,

$s_x m_x$ = average number of female offsprings
produced by an individual at age x.

Summing $s_x m_x$ across all ages, gives the average number of female offsprings produced by an individual over her reproductive lifetime hence, the Net Reproduction Number, R_0.

$$R_0 = \sum_{x=15}^{49} s_x m_x \tag{3.1}$$

Assuming that each woman produces female and male offsprings in the ratio 1 : 1 , then, $m_x = \frac{1}{2}$ of the total number of offsprings born to an individual at age x.

If we let F_0 be the average number of offsprings produced by an individual over her lifetime, then,

$$R_0 = \frac{F_0}{2} \tag{3.2}$$

$$F_0 = 2\left(\sum_{x=15}^{49} s_{x_i} m_{x_i}\right) \tag{3.3}$$

Case 1: If $F_0 = 2$, the population remains stable.

Case 2: If $F_0 < 2$, the population shrinks.

Case 3: If $F_0 > 2$, the population is increases.

Case 4: If we let ρ quantify the magnitude of the presence of interference effect, such that, $\rho \geq 1$,and that, $\rho F_0 > 2$, the population increases.

Determination of Generation time

Generation time (T) refers to the time it takes for a new born baby girl to produce a baby girl [46].

Recall; $s_x m_x$ is the average number of female offsprings born to a female at age x.

By weighting each female offspring by age of the mother 'x' when each was born , and summing across all the female offspring born in her reproductive life, we obtain $(\sum_{x=15}^{49} x s_x m_x)$. And dividing the result by $(\frac{F_o}{2})$, gives us the mean age of a female when each of her female offspring was born.

$$T = \frac{\sum_{x=15}^{49} x s_x m_x}{\sum_{x=15}^{49} s_x m_x} \tag{3.4}$$

$$T = \frac{\sum_{x=15}^{49} x s_x m_x}{\frac{1}{2} F_o} \tag{3.5}$$

$$T = \frac{2(\sum_{x=15}^{49} x s_x m_x)}{F_o} \tag{3.6}$$

Case 1 If we let $T^{'}$ be the generation time for exact replacement, $F_0 = 2$, then

$$T^{'} = \sum_{x=15}^{19} x s_x m_x \tag{3.7}$$

Case 2: If we let $T^{''}$ be the generation time for $F_0 < 2$, then, $T^{''} > T^{'}$, Generation time is therefore increased. So, it takes more time for a cohort to replace itself and the population shrinks.

Case 3: If we let $T^{'''}$ be the generation time for $F_0 > 2$, then, $T^{'''} < T^{'}$, Generation time is therefore decreased. So, hence it takes less time for a cohort to replace itself and the population increases.

Net Fertility Value and Population Growth.

R_0 represent a multiplicative factor which converts original population to a new population one generation later in the relationship $N_t = N_0(R_0)^t$, (Pianka, 2016), where, N_0 represents the population at time Zero (the present generation), and N_t represents the population after time t (later generation).

$$N_t = N_0(R_0)^t \tag{3.8}$$

$$N_t = N_0\left(\frac{F_0}{2}\right)^t \tag{3.9}$$

Case 1: - If we let $F_0 = 2$ then this implies that $N_t = N_0$, the population remains stable.
Case 2: - If we let $F_0 < 2$ then this implies that, $N_t^{'} < N_0$, the population decreases.
Case 3: - If we let $F_0 > 2$ then this implies that, $N_t^{''} > N_0$, the population increases.

CHAPTER 4

MODELING INTERFERENCE FREE DATA

4.1 Introduction

In this chapter, we modeled data sets of Rwanda 1992, Indonesia 2002 and Kenya 2003 and by using Akaikes information Criteria (AIC), we investigate among the probability distributions (Gamma, Weibull and Lognormal), the probability distribution that provides the best fit to the interference free data sets.

4.2 Rwanda 1992 fertility data model choice

Descriptive statistics and Graphical techniques were done to help identify candidate distribution functions for the Rwanda 1992 fertility data .

4.2.1 Histograms for Rwanda 1992 fertility data

The histogram for the Rwanda 1992 fertility data was plotted using R software and displayed as shown in Figure 4.1 below, (see Appendix A.1.1).

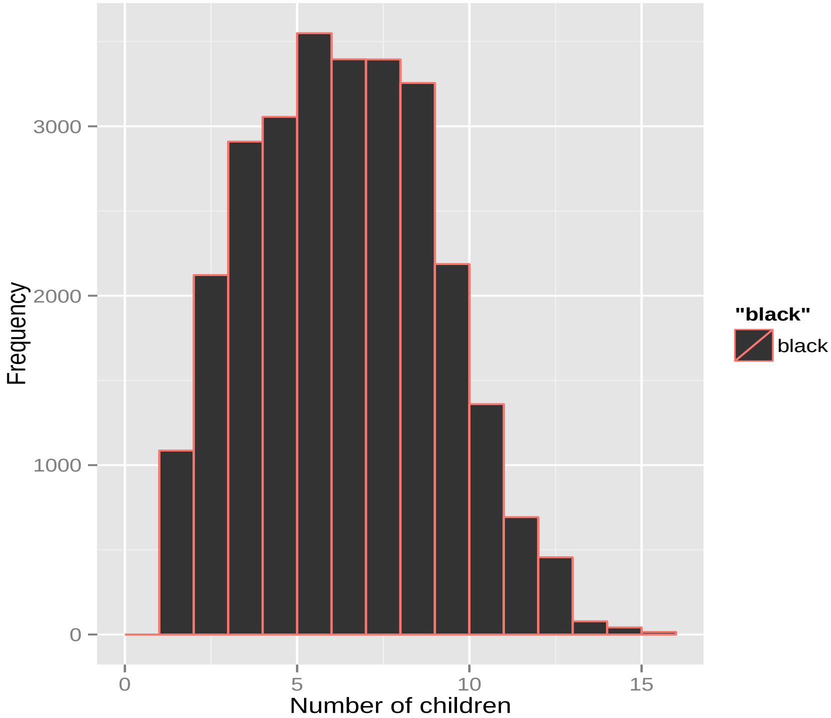

Figure 4.1: Histogram for Rwanda 1992 fertility data.

In Figure 4.1, we observe that the histogram is positively skewed. The Rwanda 1992 fertility data followed a positively skewed distribution.

4.2.2 Skewness - Kurtosis Plots for Rwanda 1992 fertility data

A skewness-kurtosis plot, proposed by Cullen and Frey in 1999 [7] was done for Rwanda 1992fertility data. The plot helps in the identification from among the positively skewed family of distributions, the candidate probability distribution that models the data set. Values of skewness and kurtosis were computed on bootstrap samples and reported as summary statistics, (see Appendix A.1.2).

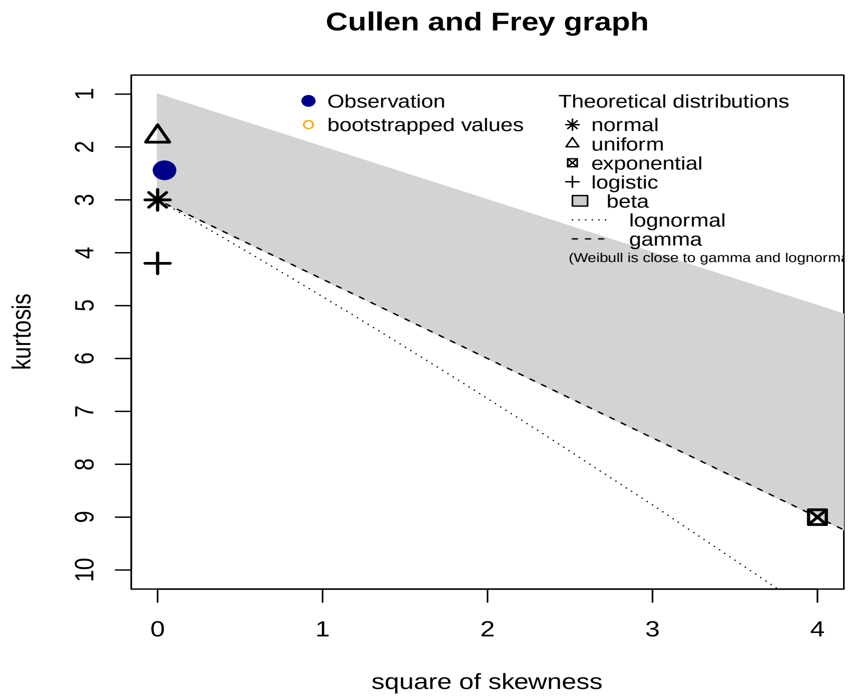

Figure 4.2: Skewness-kurtosis plot for Rwanda 1992 fertility data

```
Figure 4.2 Summary statistics
Estimated skewness:  0.2043342, Estimated kurtosis:  2.44022
```

In Figure 4.2, the skewness is non zero and positive (0.20) and kurtosis is platykurtic (2.44)The non zero skewness reveals lack of symmetry of the empirical distribution.

The kurtosis value quantifies the weight of the tails in comparison to the normal distribution for which the kurtosis equals 3.

From Figure 4.2 Summary statistics, the skewness and the kurtosis combination of the Rwanda 1992 data is $(0.20, 2.44)$. On comparing the (square of skewness, kurtosis) combination of the Rwanda 1992 data set$(0.04, 2.44)$ with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Rwanda 1992 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) combinations are all about $(0.04, 3.2)$. Our candidate models therefore were observed to be, Gamma, Lognormal and Weibull distributions.

4.2.3 Parameter estimates for Gamma, Weibull and Lognormal distributions from Rwanda 1992 data

Gamma, Weibull and Lognormal distributions parameter estimates from the Rwanda 1992 sample data were done by method of maximum likelihood using the R software. The parameter estimates from the sample data from literature gave the arbitrary values of the candidate distribution parameters that were used in the Method of MLE to fit the candidate distributions. and the results were as shown in the Table Table 4.1 below, (see Appendix A.1.3).

Table 4.1: Gamma, Weibull and Lognormal distributions Parameter estimates for Rwanda 1992 data

Country and year	Distribution	Parameter	Estimate
Rwanda 1992	Gamma	shape	4.731041
		rate	0.8039636
	Weibull	shape	3.449948
		scale	5.658448
	Lognormal	meanlog	1.6362189
		sdlog	0.5718458

From Table 4.1, the parameter estimates were then used for Rwanda 1992 data simulation purposes with respect to the three candidate distributions, which were then fitted to the simulated data by MLE. The fitted parameter estimates in Table 4.2 below were then obtained.

4.2.4 Gamma, Weibull and Lognormal distributions fits to Rwanda 1992 fertility data

The parameters of the fitted models were estimated by MLE with the help of R software. The numerical results, returned by the software were; the fitted parameter estimates (after the iterative procedure) and the Akaike's Information Criteria (AIC), (see appendix A.1.4).

Table 4.2: Fitted model parameter estimates for Gamma, Weibull and Lognormal to Rwanda 1992 data

Distribution	Parameter	Estimate	AIC
Gamma	shape	4.7675508	128990.5
	rate	0.8102551	
Weibull	shape	2.316357	130270.5
	scale	6.653763	
Lognormal	meanlog	1.6637720	130039.1
	sdlog	0.4832737	

From Table 4.2 above, we observed that Gamma distribution fitted with the lowest AIC value as compared to Weibull and Lognormal distribution. Gamma distribution looses the least amount of information in fitting the Rwanda 1992 data set, as evidenced by the lowest AIC value that the Gamma parameters recorded. Gamma distribution is therefore a better model for the Rwanda 1992 data set as compared to Weibull and Lognormal distributions.

4.2.5 Quality and Goodness of fit test for Model to Rwanda 1992 fertility data

The density functions of the Gamma, Weibull and Lognormal and the histogram of Rwanda 1992 were plotted and given in Figure 4.3 below, (see Appendix A.1.5).

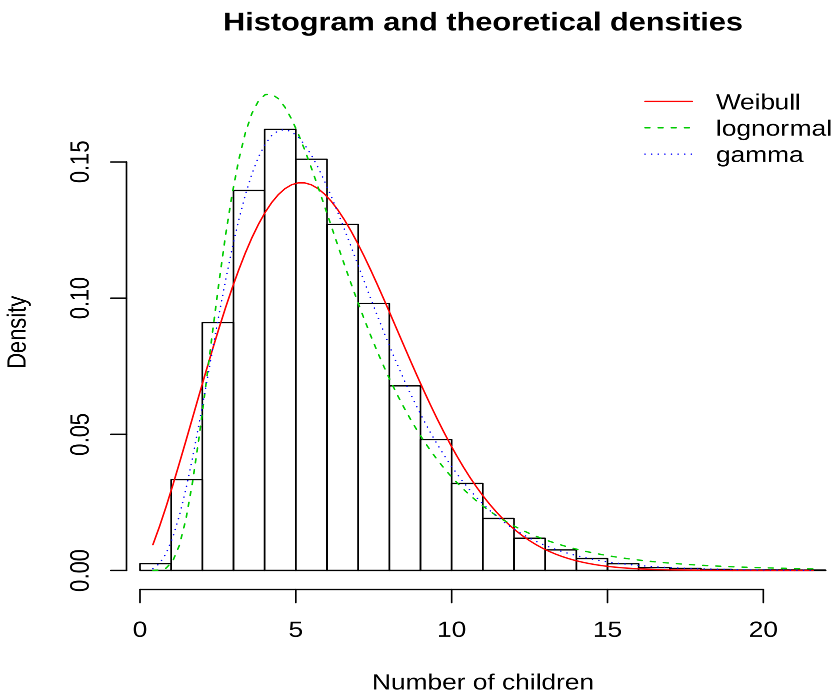

Figure 4.3: Density plots of some distributions on Histogram for Rwanda 1992 fertility data.

From Figure 4.3 above, we see that gamma distribution mentioned earlier fits the data best.

The quantile - quantile $(Q-Q)$ plot as shown in Figure 4.4 below was also done to give a graphical technique for testing for the goodness of fit for each model to the Rwanda 1992 fertility data.

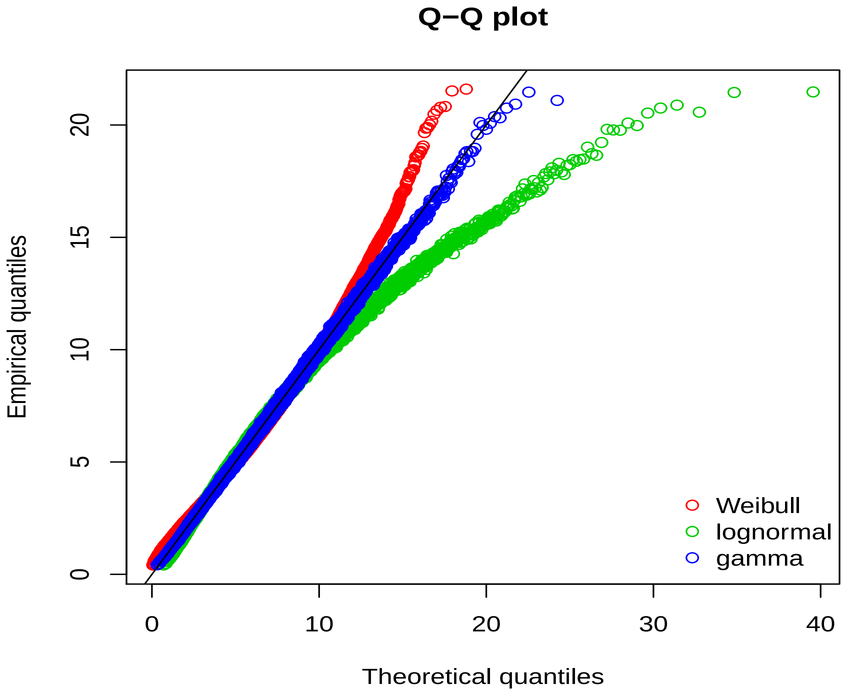

Figure 4.4: Q-Q plots for Gamma, Weibull and Lognormal for Rwanda 1992 fertility data.

In Figure 4.4, most of the Gamma points lie along the empirical line hence Gamma distribution is a better model for the Rwanda 1992 as compared to the Weibull and the Lognormal distributions.

4.3 Indonesia 2002 fertility data model choice

Descriptive statistics and Graphical techniques were used to identify the candidate distribution for the Indonesia 2002 fertility data.

4.3.1 Histograms for Indonesia 2002 fertility data

The histogram for the Indonesia 2002 fertility data was plotted using R software and displayed as shown below, (see Appendix A.2.1).

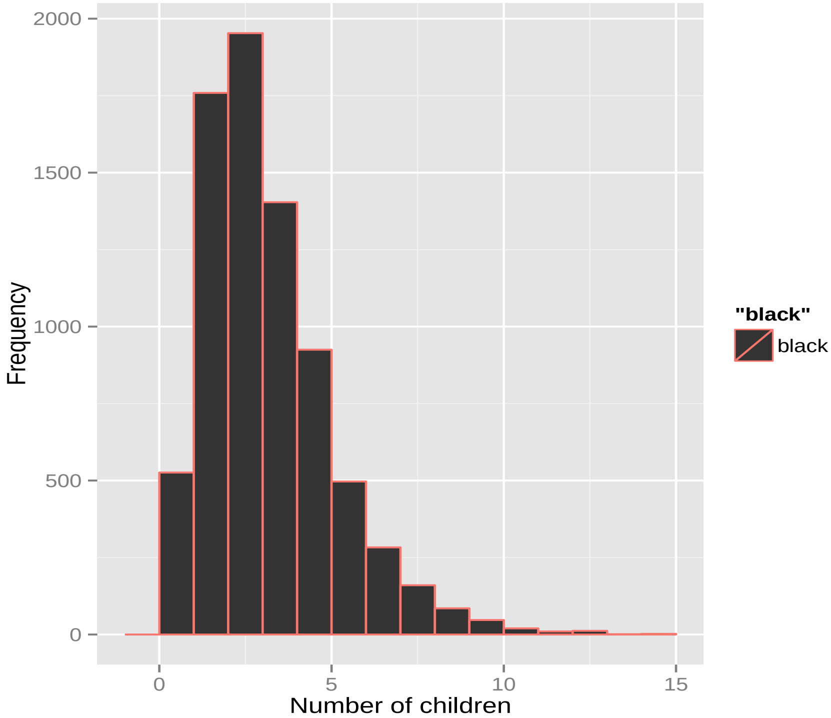

Figure 4.5: Histogram for Indonesia 2002 fertility data.

From Figure 4.5, histogram was positively skewed hence, candidate models for Indonesia 2002 fertility data were positively skewed.

4.3.2 Skewness - Kurtosis Plots for Indonesia 2002 fertility data

A skewness-kurtosis plot was done for Indonesia 2002 and positively skewed candidate distributions identified. Values of skewness and kurtosis were computed on bootstrap samples and reported as summary statistics, (see Appendix A.2.2).

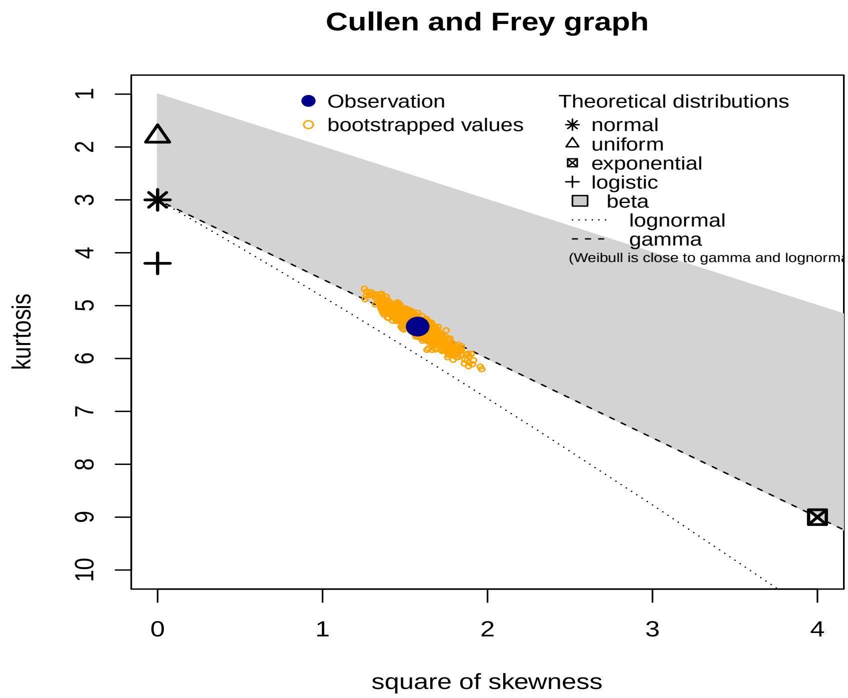

Figure 4.6: Skewness-kurtosis plot for Indonesia 2002 fertility data.

```
Figure 4.6 summary statistics
Estimated skewness:  1.255609, Estimated kurtosis:  5.395964
```

Figure 4.6 shows that the skewness is non Zero and positive (1.26) and kurtosis is leptokurtic (5.40) for the Indonesia 2002 data. The non zero skewness from the graph reveals lack of symmetry of the empirical distribution.

From the Cullen and Frey graph (Figure 4.6), the skewness and the kurtosis combination of the Indonesia 2002 data is $(1.26, 5.40)$. On comparing the (square of skewness, kurtosis) combination of the Indonesia 2002 data set$(1.58, 5.40)$ with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Indonesia 2002 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) were about $(1.58, 5.2)$, $(1.58, 5.5)$ and $(1.58, 6.0)$ respectively. Our candidate models therefore, were Gamma, Lognormal and Weibull.

4.3.3 Parameter estimates for Gamma, Weibull and Lognormal distributions from Indonesia 2002 data

Gamma, Weibull and Lognormal distributions parameter estimates from the Indonesia 2002 sample data were done by method of maximum likelihood and the values shown in Table 4.3 below, (see Appendix A.2.3).

Table 4.3: Gamma, Weibull and Lognormal distributions
Parameter estimates for Indonesia 2002 data

Distribution	Parameter	Estimate
Gamma	shape	1.974798
	rate	0.7410074
Weibull	shape	0.8856127
	scale	4.229692
Lognormal	meanlog	0.7045324
	sdlog	0.7994178

From Table 4.3, the parameter estimates were then used for Indonesia 2002 data simulation purposes with respect to the three candidate distributions, which were then fitted to the simulated data by MLE. The fitted parameter estimates in Table 4.3 below were then obtained.

4.3.4 Gamma, Weibull and Lognormal distributions fits to Indonesia 2002 fertility data

The parameters of the fitted models were estimated by maximum likelihood method using R software and the parameter estimates and the Akaike's Information Criteria (AIC) determined as shown in Table 4.4 below, (see Appendix A.2.4).

Table 4.4: Fitted model Parameter estimates for Gamma, Weibull and Lognormal distributions to Indonesia 2002 fertility data

Distribution	parameter	estimate	AIC
Gamma	shape	2.0218812	28476.32
	rate	0.7651225	
Weibull	shape	1.492607	28574.74
	scale	2.934566	
Lognormal	meanlog	0.7045505	29223.92
	sdlog	0.8008027	

From Table 4.4, Gamma distribution fitted with the lowest AIC value. Gamma distribution therefore lost the least information when used to generate the Indonesia 2002 fertility data set hence was a better model for the data compared to Weibull or Lognormal.

4.3.5 Quality and Goodness of fit test for Model to Indonesia 2002 fertility data

The density functions of the Gamma, Weibull and Lognormal and the histogram of Indonesia 2002 were plotted and given in Figure 4.7 below for the quality of fit assessment, (see Appendix A.2.5).

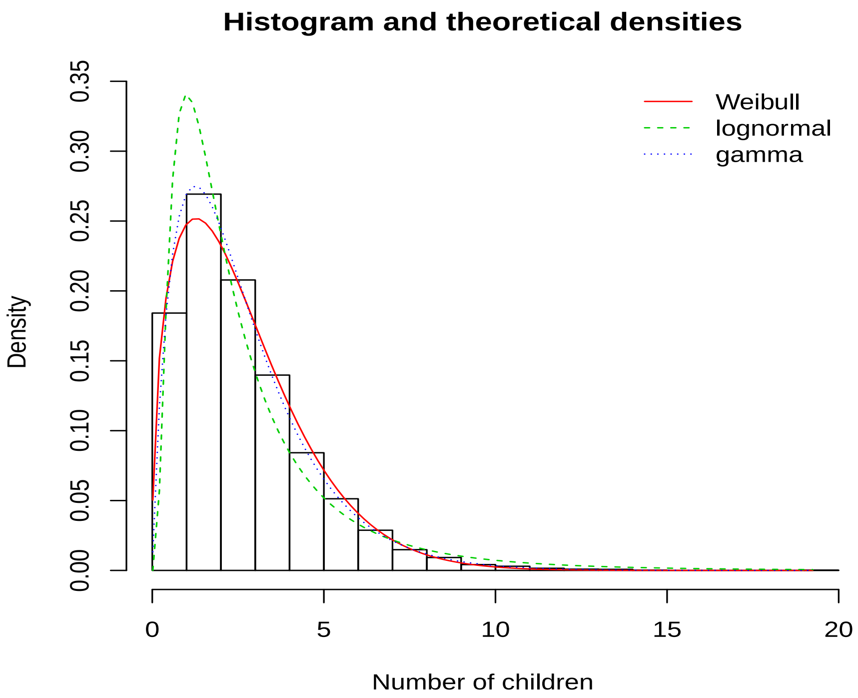

Figure 4.7: Density plots of some distributions on Histogram for Indonesia 2002 fertility data.

From Figure 4.7, Gamma distribution as mentioned earlier fits the data best.

The $Q-Q$ plots for Gamma, Weibull and Lognormal for Indonesia 2002 were plotted in Figure 4.8 below for graphical testing for the goodness of fit.

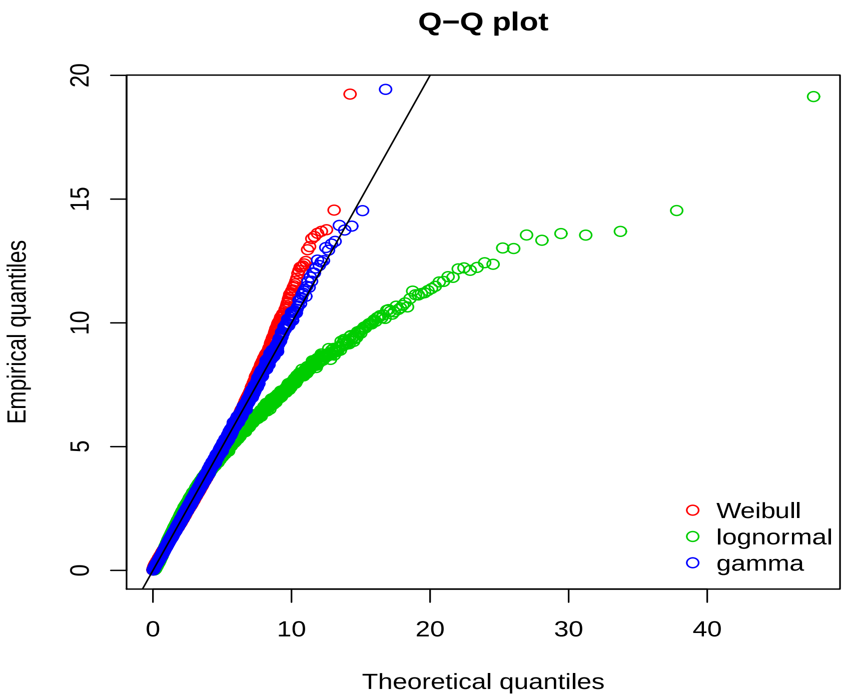

Figure 4.8: Q-Q plots for Gamma, Weibull and Lognormal for Indonesia 2002 fertility data.

Gamma distribution fits the data best to the Indonesia 2002 fertility data as most of the Gamma point lied along the empirical straight line. Gamma distribution was observed to be the best fitting model for the Indonesia 2002 fertility data.

4.4 Kenya 2003 fertility data model choice

Descriptive statistics and Graphical techniques were used to identify the candidate distribution for the Kenya 2003 fertility data .

4.4.1 Histograms for Kenya 2003 fertility data

The histogram for the Kenya 2003 fertility data was plotted using R software and displayed as shown below, (see Appendix A.3.1).

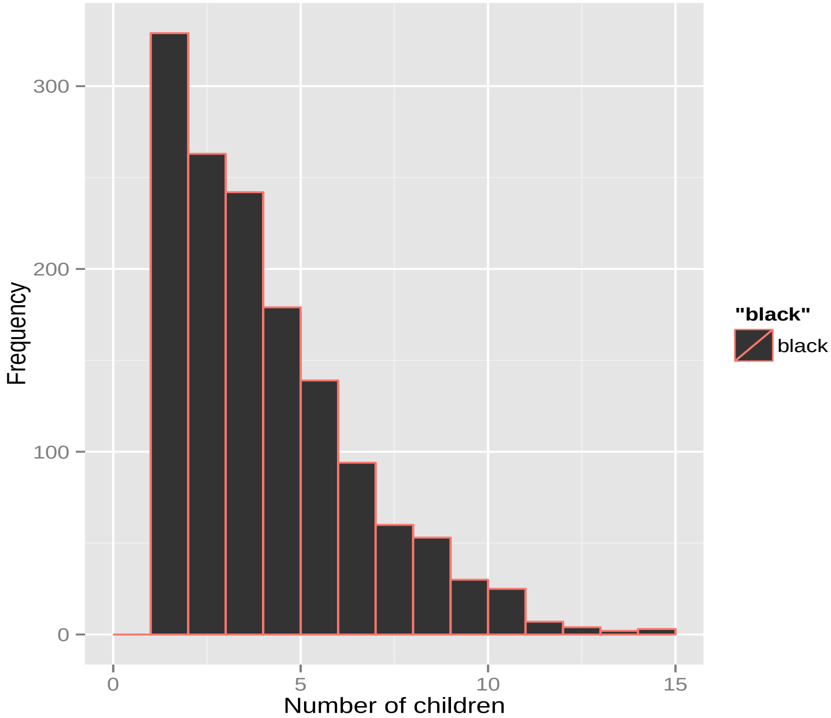

Figure 4.9: Histogram for Kenya 2003 fertility data.

Figure 4.9 above shows that the histogram is positively skewed. The candidate distributions for the Kenya 2003 fertility data are positively skewed.

4.4.2 Skewness - Kurtosis Plots for Kenya 2003 fertility data

A skewness-kurtosis plot [7] was done for Kenya 2003 and positively skewed candidate distributions identified. (see Appendix A.3.2).

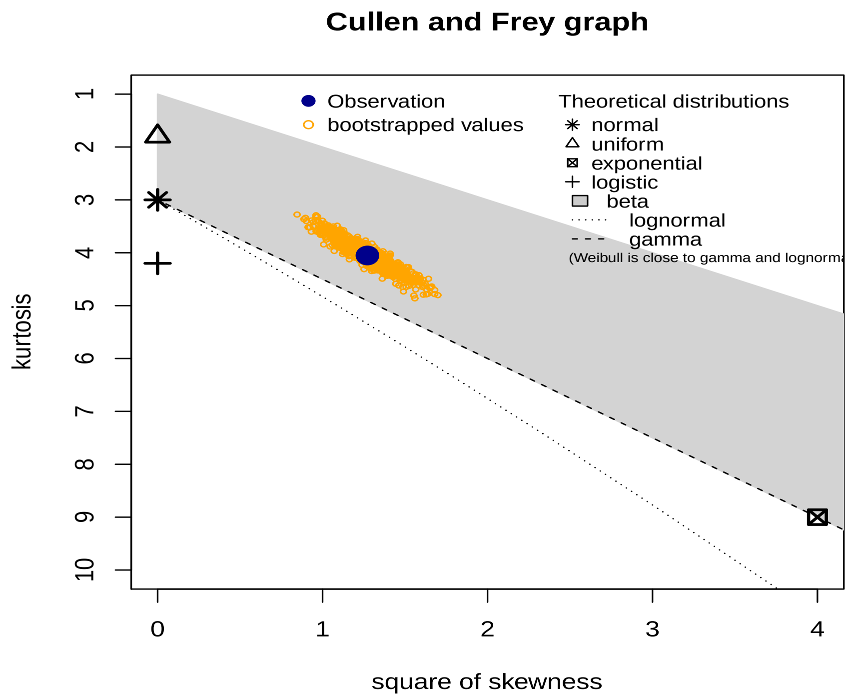

Figure 4.10: Skewness-kurtosis plot for Kenya 2003 fertility data

```
Figure 4.10 Summary Statistics
Estimated skewness:  1.12774, Estimated kurtosis:  4.051652
```

Figure 4.10 shows that the skewness is non Zero and positive (1.13) and kurtosis is leptokurtic (4.05) for the Kenya 2003 data. The non zero skewness from the graph reveals lack of symmetry of the empirical distribution.

From the Cullen and Frey graph (Figure 4.10) the skewness and the kurtosis combination of the Kenya 2003 data is $(1.13, 4.05)$. On comparing the (square of skewness, kurtosis) combination of the Kenya 2003 data set($(1.28, 4.05)$) with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Kenya 2003 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) were about $(1.28, 4.8)$, $(1.28, 5.0)$ and $(1.28, 5.2)$ respectively. Our candidate models therefore, were Gamma, Lognormal and Weibull.

4.4.3 Parameter estimates for Gamma, Weibull and Lognormal distributions from Kenya 2003 data

Gamma, Weibull and Lognormal distributions parameter estimates from the Kenya 2003 sample data was done by method of maximum likelihood and the values are as shown in Table 4.5 below, (see Appendix A.3.3).

Table 4.5: Gamma, Weibull and Lognormal distributions parameter estimates for Kenya 2003 data

Distribution	Parameter	Value
Gamma	shape	1.908814
	rate	0.5442879
Weibull	shape	1.168336
	scale	4.700725
Lognormal	meanlog	0.9949621
	sdlog	0.8031937

From Table 4.5, the parameter estimates were then used for Kenya 2003 data simulation purposes with respect to the three candidate distributions, which were then fitted to the simulated data by MLE. The fitted parameter estimates in Table 4.6 below were then obtained.

4.4.4 Gamma, Weibull and Lognormal distributions fits to Kenya 2003 fertility data

The parameters of the fitted models were estimated by maximum likelihood method using R software and the parameter estimates and the Akaike's Information Criteria (AIC) determined as shown in Table 4.6 below, (see Appendix A.3.4).

Table 4.6: Fitted model Parameter estimates for Gamma, Weibull and Lognormal distributions to Kenya 2003 fertility data

Distribution	parameter	estimate	AIC
Gamma	shape	1.9894204	6070.594
	rate	0.5782404	
Weibull	shape	1.486736	6084.772
	scale	3.818015	
Lognormal	meanlog	0.9636709	6201.48
	sdlog	0.8060106	

From Table 4.6, Gamma distribution fitted with the lowest AIC value. Gamma distribution therefore lost the least information when used to generate the Kenya 2003 fertility data set hence was a better model for the data compared to Weibull or Lognormal.

4.4.5 Quality and Goodness of fit test for Model to Kenya 2003 fertility data

The density functions of the Gamma, Weibull and Lognormal and the histogram of Kenya 2003 were plotted and given in the Figure 4.11 below for the quality of fit assessment, (see Appendix A.3.5).

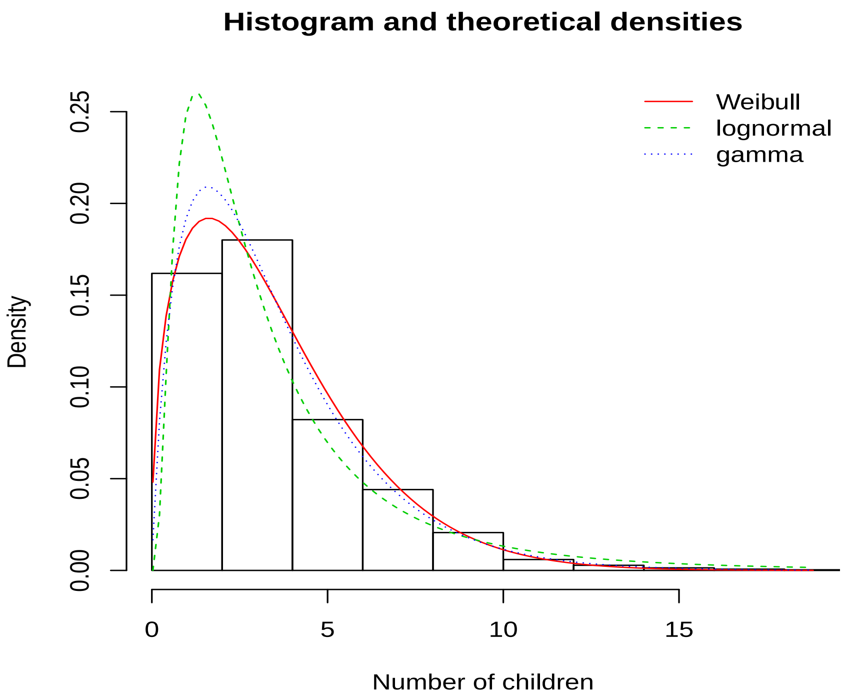

Figure 4.11: Density plots of some distributions on Histogram for Kenya 2003 fertility data

From Figure 4.11, Gamma distribution as mentioned earlier fits the data best.

The $Q-Q$ plots for Gamma, Weibull and Lognormal for Kenya 2003 were plotted in Figure 4.12 below for graphical testing for the goodness of fit.

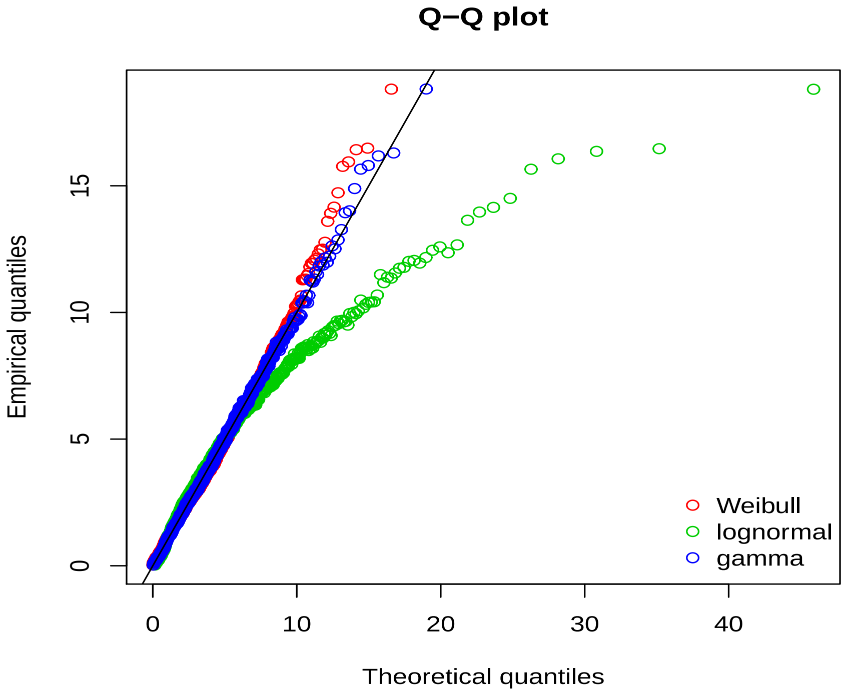

Figure 4.12: Q-Q plots for Gamma, Weibull and Lognormal for Kenya 2003 fertility data.

Gamma distribution fits the data best to the Kenya 2003 fertility data as most of the Gamma point lied along the empirical straight line. Gamma distribution was observed to be the best fitting model for the Kenya 2003 fertility data.

CHAPTER 5

MODELING DATA CONTAINING INTERFERENCE EFFECT

5.1 Introduction

In this chapter, we model the data sets of Rwanda 2000, Indonesia 2007 and Kenya 2009 which contain the effect of interference. Using Akaikes information Criteria (AIC), we investigated among the probability distributions (Gamma, Weibull and Lognormal), the distribution that best fits the data sets containing the effect of interference.

5.2 Rwanda 2000 fertility data model choice

Descriptive statistics and Graphical techniques were done to help identify candidate distribution functions for the Rwanda 2000 fertility data .

5.2.1 Histogram for Rwanda 2000 fertility data

The histogram for the Rwanda 2000 fertility data was plotted using R software and displayed as shown below, (see Appendix B.1.1).

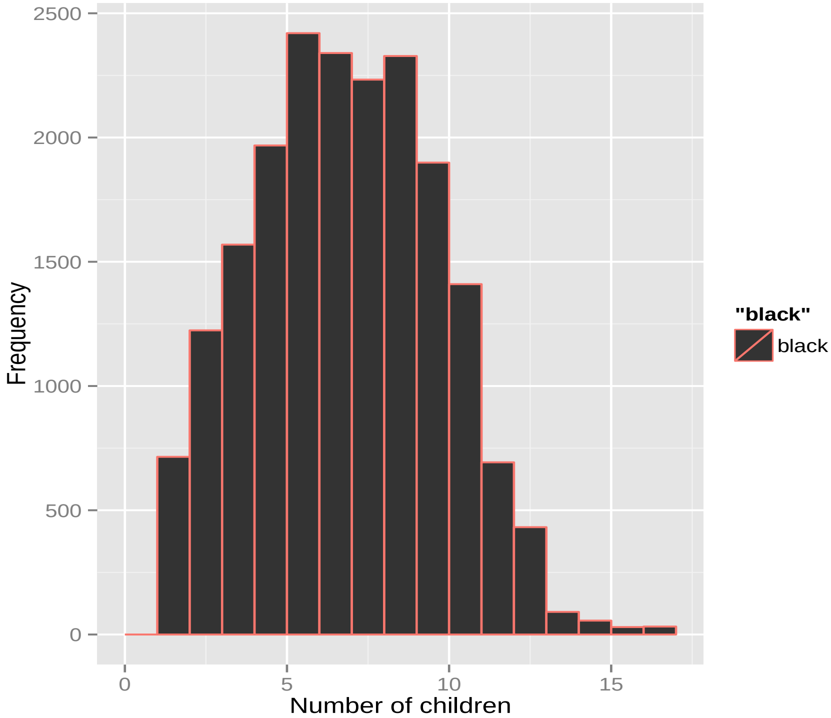

Figure 5.1: Histogram for Rwanda 2000 fertility data.

From Figure 5.1 above, the histogram was observed to be positively skewed. The candidate distributions were therefore positively skewed.

5.2.2 Skewness - Kurtosis Plots for Rwanda 2000 fertility data

A skewness-kurtosis plot, proposed by Cullen and Frey in 1999 [7], and provided by the 'descdist' function in the R software was done. Values of skewness and kurtosis were computed on bootstrap samples and reported on skewness-kurtosis plot as shown below, (see Appendix B.1.2).

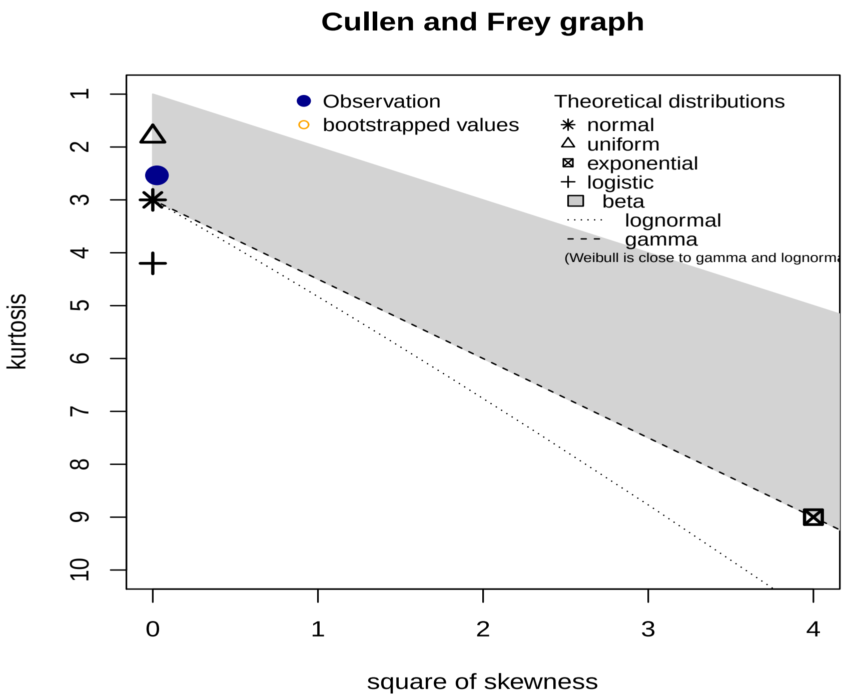

Figure 5.2: Skewness-kurtosis plot for Rwanda 2000 fertility data

```
Figure 5.2 summary statistics
Estimated skewness:  0.1583683, Estimated kurtosis:  2.536246
```

The non zero skewness from the graph reveals lack of symmetry of the empirical distribution.

The kurtosis value quantifies the weight of the tails in comparison to the normal distribution for which the kurtosis equals 3.

From the Cullen and Frey graph, the skewness and the kurtosis combination of the Rwanda 2000 data is $(0.16, 2.54)$. On comparing the (square of skewness, kurtosis) combination of the Rwanda 2000 data set$(0.03, 4.05)$ with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Rwanda 2000 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) all were about $(0.03, 3.1)$. The candidate models therefore, were Gamma, a Lognormal and Weibull.

5.2.3 Parameter estimates for Gamma, Weibull and Lognormal distributions from Rwanda 2000 data

Gamma, Weibull and Lognormal distributions parameter estimates from the Rwanda 2000 sample data was done by method of maximum likelihood using the R software and the results are shown in Table 5.1 below, (see Appendix B.1.3).

Table 5.1: Gamma, Weibull and Lognormal distributions
Parameter estimates for Rwanda 2000 data

Distribution	Parameter	Value
Gamma	shape	4.946252
	rate	0.8113425
Weibull	shape	3.658164
	scale	5.996006
Lognormal	meanlog	1.7299596
	sdlog	0.4728956

From Table 5.1, the parameter estimates were then used for Rwanda 2000 data simulation purposes with respect to the three candidate distributions, which were then fitted to the simulated data by MLE. The fitted parameter estimates in Table 5.2 below were then obtained.

5.2.4 Gamma, Weibull and Lognormal distributions fits to Rwanda 2000 fertility data

The parameters of the fitted models were estimated by maximum likelihood using R software.

The numerical results, returned by the software were; the parameter estimates and the Akaike's Information Criteria (AIC), (see Appendix B.1.4).

Table 5.2: Fitted model parameter estimates for Gamma, Weibull and Lognormal to Rwanda 2000 data

Distribution	parameter	estimate	AIC
Gamma	shape	4.9727711	92923.67
	rate	0.8230042	
Weibull	shape	2.354821	93950.42
	scale	7.142197	
Lognormal	meanlog	1.5300374	93544.51
	sdlog	0.4711724	

From Table 5.2 above, we see that Gamma distribution fitted with the lowest AIC value, hence a better model for the Rwanda 2000 data set than Weibull and Lognormal distributions.

5.2.5 Quality and Goodness of fit test for Model to Rwanda 2000 fertility data

The density functions of the Gamma, Weibull and Lognormal and the histogram of Rwanda 2000 were plotted as shown in Figure 5.3 below, (see Appendix B.1.5).

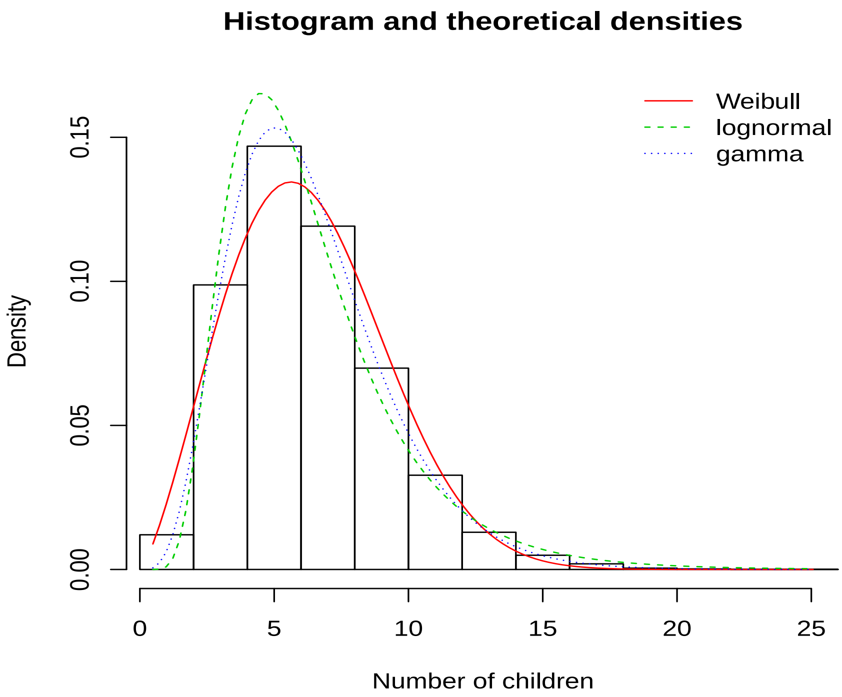

Figure 5.3: Density plots of some distributions on Histogram for Rwanda 2000 fertility data.

From Figure 5.3 above, we see that gamma as given earlier fits the data best.

The goodness of fit of the quantile - quantile plot for the comparison of the fitted models and the empirical distributions was also done to assess the quality of fit of the models to Rwanda 2000 data, and the result displayed in Figure 5.4 below .

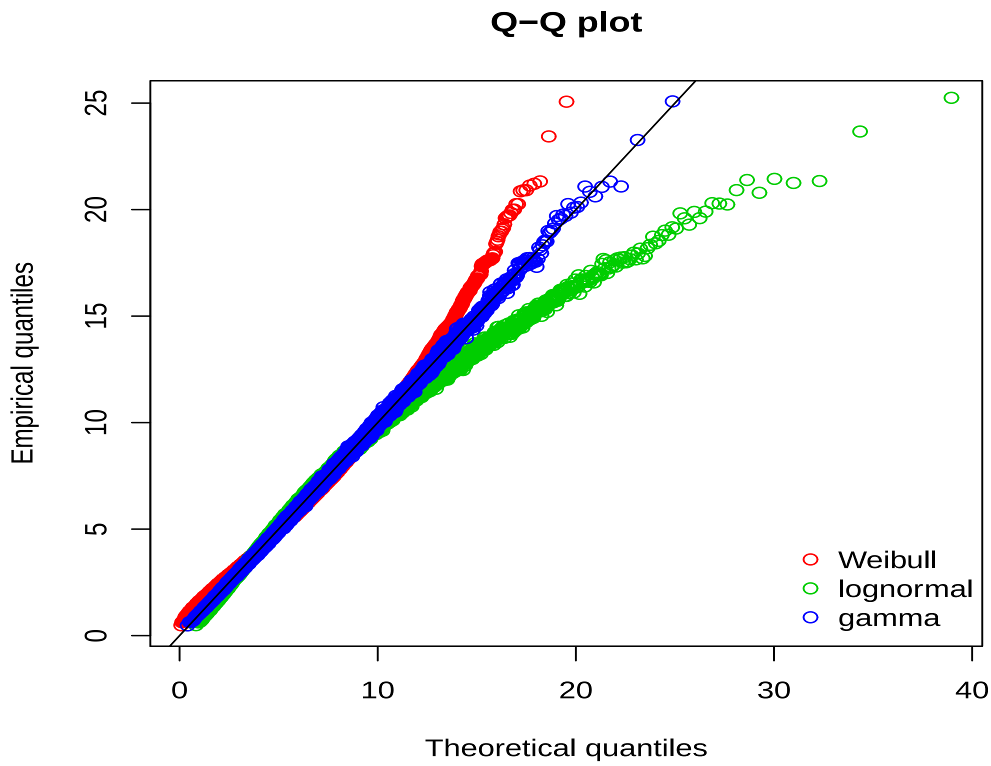

Figure 5.4: Q-Q plot for Gamma, Weibull and Lognormal for Rwanda 2000 fertility data.

From Figure 5.4, Gamma distribution fits the data better than the Weibull and the Lognormal distributions.

5.3 Indonesia 2007 fertility data model choice

Exploratory data analysis by the use of descriptive statistics and Graphical techniques were done to help identify candidate distribution functions for the Indonesia 2007 fertility data .

5.3.1 Histograms for Indonesia 2007 fertility data

The histogram for the Indonesia 2007 fertility data was plotted using R software and displayed as shown below, (see Appendix B.2.1).

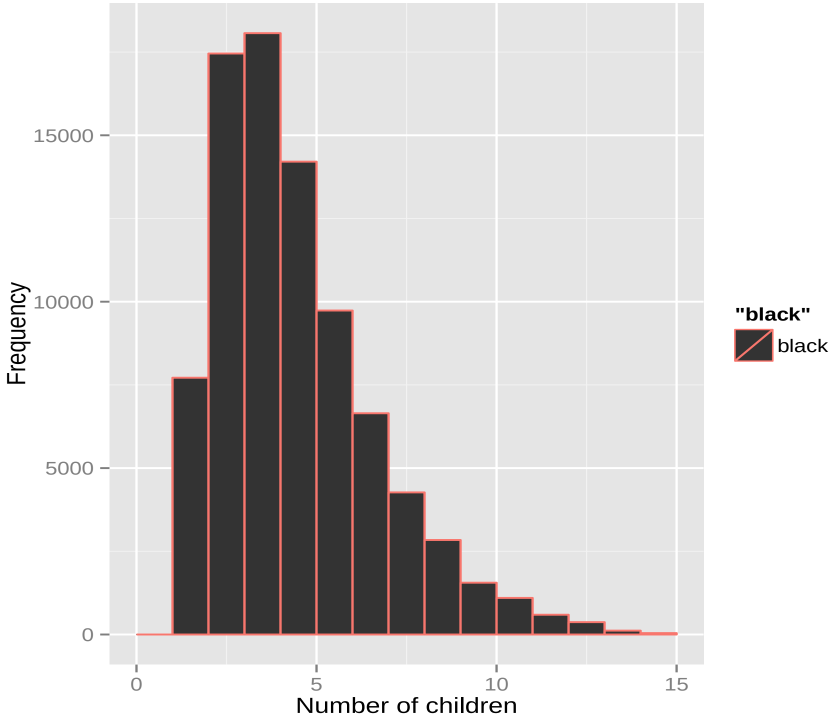

Figure 5.5: Histogram for Indonesia 2007 fertility data

In Figure 5.5 above, we observe that the Histogram is positively skewed. The Indonesia 2007 fertility data was positively skewed hence candidate distributions were from a family of positively skewed distributions.

5.3.2 Skewness - Kurtosis Plots for Indonesia 2007 fertility data

A skewness-kurtosis plot, proposed by Cullen and Frey in 1999 [7], and provided by the 'descdist' function in the R software was done. Values of skewness and kurtosis were computed on bootstrap samples and were reported on skewness-kurtosis plot as shown in Figure 5.6 below, (see Appendix B.2.2).

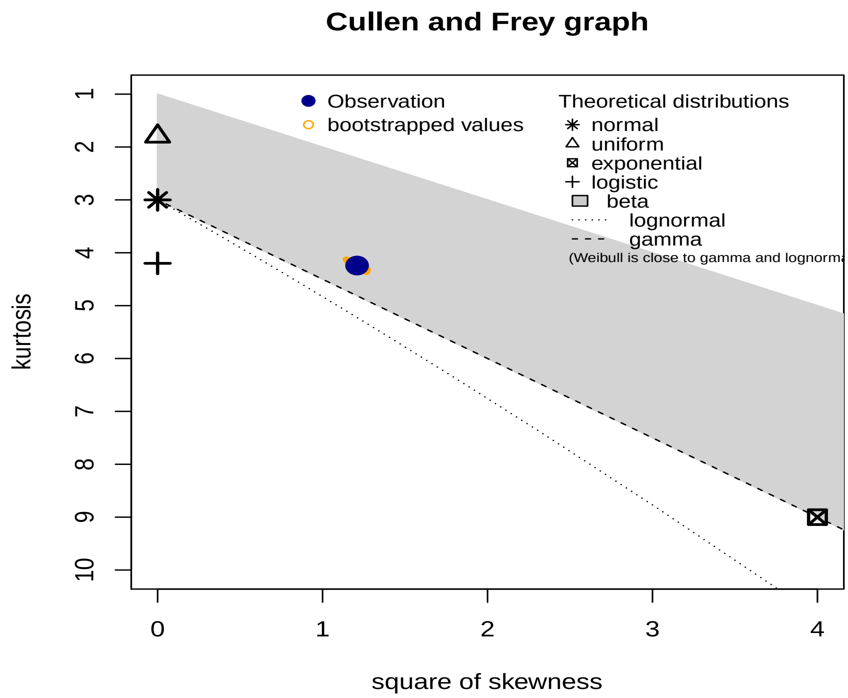

Figure 5.6: Skewness - Kurtosis Plots for Indonesia 2007 fertility data

```
Figure 5.6 summary statistics
Estimated skewness:  1.099586, Estimated kurtosis:  4.241901
```

The non zero skewness from the graph reveals lack of symmetry of the empirical distribution.

The kurtosis value quantifies the weight of the tails in comparison to the normal distribution for which the kurtosis equals 3.

From the Cullen and Frey graph, the skewness and the kurtosis combination of the Indonesia 2007 data is $(1.10, 4.24)$. On comparing the (square of skewness, kurtosis) combination of the Indonesia 2007 data set($(1.21, 4.24)$) with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Indonesia 2007 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) were about $(1.21, 4.7)$, $(1.21, 4.9)$ and $(1.21, 5.1)$ respectively. Our candidate models therefore, were Gamma, Lognormal and Weibull.

5.3.3 Parameter estimates for Gamma, Weibull and Lognormal distributions from Indonesia 2007 data

Gamma, Weibull and Lognormal distributions parameter estimates from the Indonesia 2002 sample data was done by method of maximum likelihood using the R software and the results are given in tables 4.1 below, (see Appendix B.2.3).

Table 5.3: Gamma, Weibull and Lognormal distributions
Parameter estimates for Indonesia 2007 data

Distribution	Parameter	Value
Gamma	shape	3.09893
	rate	0.7885535
Weibull	shape	1.939006
	scale	4.038839
Lognormal	meanlog	1.1958164
	sdlog	0.6176233

From Table 5.3, the parameter estimates were then used for Indonesia 2007 data simulation purposes with respect to the three candidate distributions, which were then fitted to the simulated data by MLE. The fitted parameter estimates in Table 5.4 below were then obtained.

5.3.4 Gamma, Weibull and Lognormal distributions fits to Indonesia 2007 fertility data

The parameters of the fitted models were estimated by maximum likelihood method available in the ‘fitdistrplus’ in the R software.
The numerical results, returned by the software were; the parameter estimates and the Akaike’s Information Criteria (AIC), (see Appendix B.2.4).

Table 5.4: Fitted model Parameter estimates for Gamma, Weibull and Lognormal distributions to Indonesia 2007 fertility data

Distribution	parameter	estimate	AIC
Gamma	shape	3.0914084	356450.6
	rate	0.7888381	
Weibull	shape	1.860316	359050.4
	scale	4.428022	
Lognormal	meanlog	1.1954818	361750.3
	sdlog	0.6165636	

From Table 5.4 above, Gamma distribution fitted with the lowest AIC value, which means that Gamma distribution fits the data better than Weibull and Lognormal distributions.

5.3.5 Quality and Goodness of fit test for Model to Indonesia 2007 fertility data

The density functions of the Gamma, Weibull and Lognormal and the histogram of Indonesia 2007 were plotted and given in Figure 5.7 below, (see Appendix B.2.5).

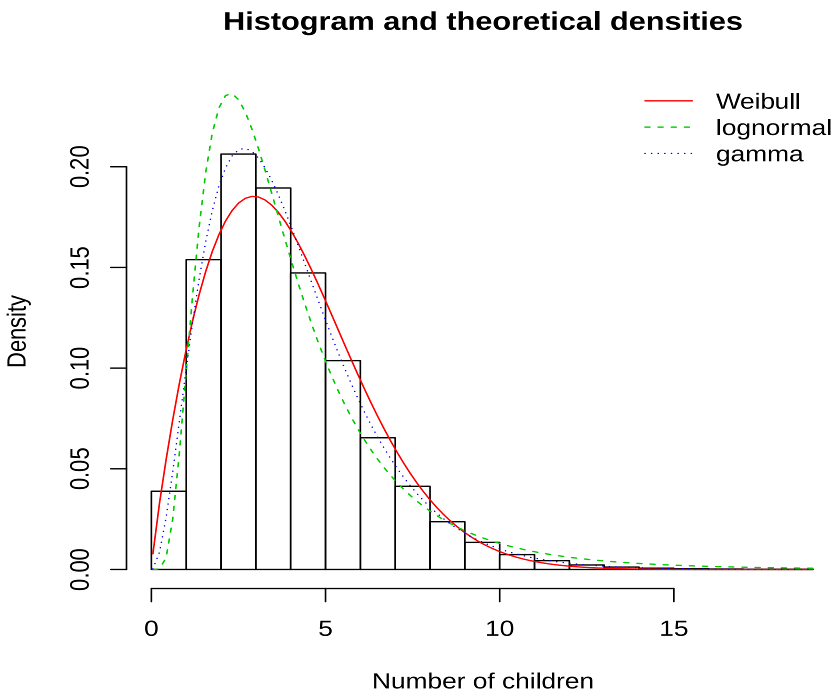

Figure 5.7: Density plots of some distributions on Histogram for Indonesia 2007 fertility data

From Figure 5.7 we see that gamma as given earlier fits the data best.

To assess the quality of fit, quantile-quantile plot for the comparison of the Gamma, Weibull and Lognormal fitted models and the empirical distribution were also done and the result of this displayed in Figure 5.8 below .

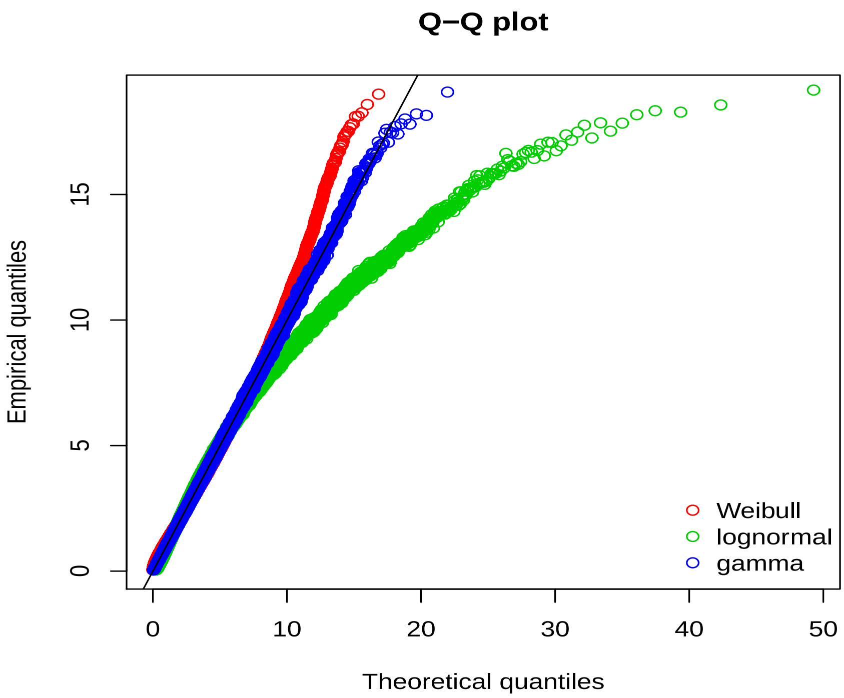

Figure 5.8: Q-Q plots for Gamma, Weibull and Lognormal for Indonesia 2007 fertility data.

From Figure 5.8, most of the Gamma points lied along the empirical straight line. Therefore, Gamma distribution fitted the Indonesia 2007 fertility data better than Weibull and Lognormal.

5.4 Kenya 2009 fertility data model choice

Exploratory data analysis by the use of descriptive statistics and Graphical techniques were done to help identify candidate distribution functions for the Kenya 2009 fertility data.

5.4.1 Histograms for Kenya 2009 fertility data

The histogram for the Kenya 2009 fertility data was plotted using R software and displayed as in Figure 5.9 below, (see appendix B.3.1).

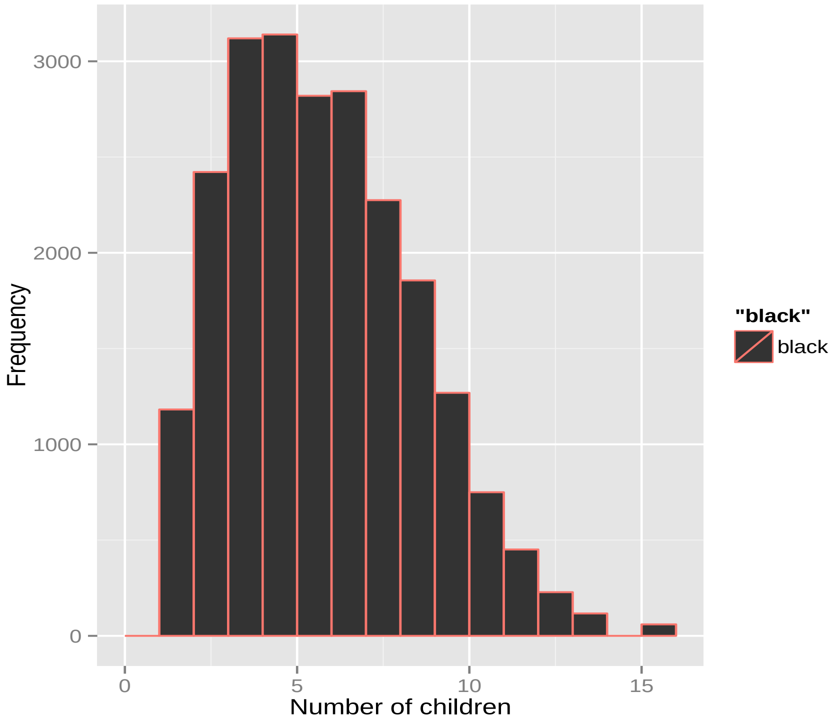

Figure 5.9: Histogram for Kenya 2009 fertility data.

From Figure 5.9, the Histogram was positively skewed. Candidate distributions were therefore, positively skewed.

5.4.2 Skewness - Kurtosis Plots for Kenya 2009 fertility data

A skewness-kurtosis plot, [7], was done. Values of skewness and kurtosis were computed on bootstrap samples and reported on skewness-kurtosis plot as shown below, (see Appendix B.3.2).

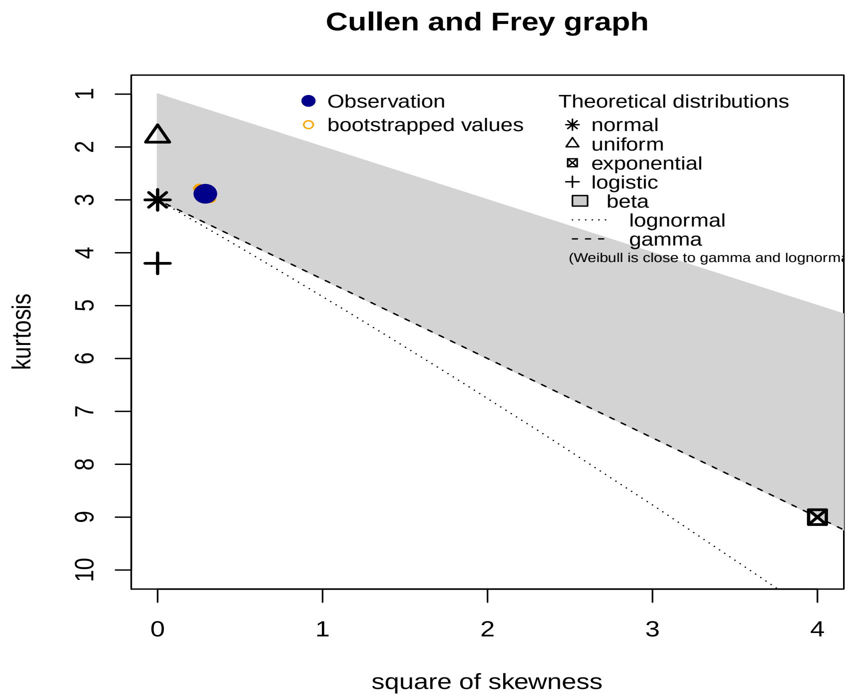

Figure 5.10: Skewness-kurtosis plot for Kenya 2009 fertility data

```
Figure 5.10 summary statistics
Estimated skewness:  0.5378432, Estimated kurtosis:  2.884296
```

The non zero skewness from the graph reveals lack of symmetry of the empirical distribution.

The kurtosis value quantifies the weight of the tails in comparison to the normal distribution for which the kurtosis equals 3.

From the Cullen and Frey graph, the skewness and the kurtosis combination of the Kenya 2009 data is $(0.54, 2.88)$. On comparing the (square of skewness, kurtosis) combination of the Kenya 2009 data set$(0.29, 2.88)$ with the (square of skewness, kurtosis) combinations that can be assumed by other distributions, we observe a consistency of the Kenya 2009 data with the Gamma, Weibull and Lognormal distributions whose (square of skewness, kurtosis) were about $(0.29, 3.3)$, $(0.29, 3.4)$ and $(0.29, 3.5)$ respectively. Our candidate models therefore, were Gamma, Lognormal and Weibull.

5.4.3 Parameter estimates for Gamma, Weibull and Lognormal distributions from Kenya 2009 data

Gamma, Weibull and Lognormal distributions parameter estimates from the Kenya 2009 sample data was done by method of maximum likelihood using the R software and the results were reported in Table 5.5 below, (see Appendix B.3.3).

Table 5.5: Gamma, Weibull and Lognormal distributions
Parameter estimates for Kenya 2009 data

Distribution	Parameter	Value
Gamma	shape	3.845601
	rate	0.7286859
Weibull	shape	2.826407
	scale	5.143644
Lognormal	meanlog	1.5335975
	sdlog	0.5425868

From Table 5.5, the parameter estimates were then used for Kenya 2009 data simulation purposes with respect to the three candidate distributions, which were then fitted to the simulated data by MLE. The fitted parameter estimates in Table 5.6 below were then obtained.

5.4.4 Gamma, Weibull and Lognormal distributions fits to Kenya 2009 fertility data

The parameters of the fitted models were estimated by maximum likelihood method using R software.
The numerical results, returned by the software were; the parameter estimates and the Akaike's Information Criteria (AIC), (see Appendix B.3.4).

Table 5.6: Fitted model Parameter estimates for Gamma, Weibull and Lognormal distributions to Kenya 2003 fertility data

Distribution	parameter	estimate	AIC
Gamma	shape	3.7706601	104686.9
	rate	0.7155335	
Weibull	shape	2.059387	105522.9
	scale	5.964338	
Lognormal	meanlog	1.5235397	105776.3
	sdlog	0.5512704	

From Table 5.6 above, we see that Gamma distribution fitted with the lowest AIC value, hence Gamma distribution fitted the data better than Weibull and Lognormal distributions.

5.4.5 Quality and Goodness of fit test for Model to Kenya 2009 fertility data

The density functions of the Gamma, Weibull and Lognormal and the histogram of Kenya 2009 were plotted as shown in Figure 5.11 below, (see Appendix B.3.5).

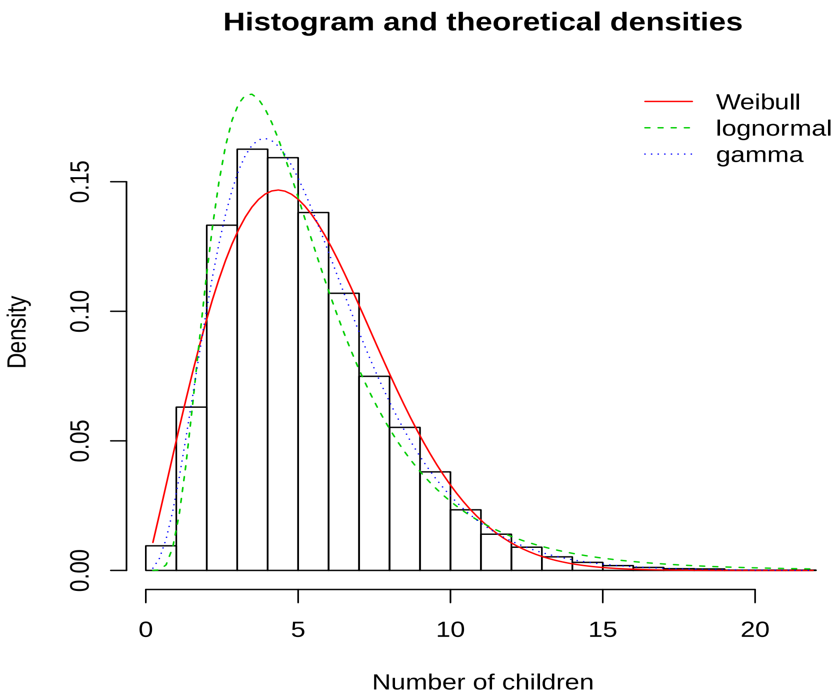

Figure 5.11: Density plots of some distributions on Histogram for Kenya 2009 fertility data

From Figure 5.11 above, Gamma as given earlier fits the data best.

The goodness of fit of the quantile-quantile plot for the comparison of the fitted models and the empirical distributions was also done and the result displayed in Figure 5.12 below .

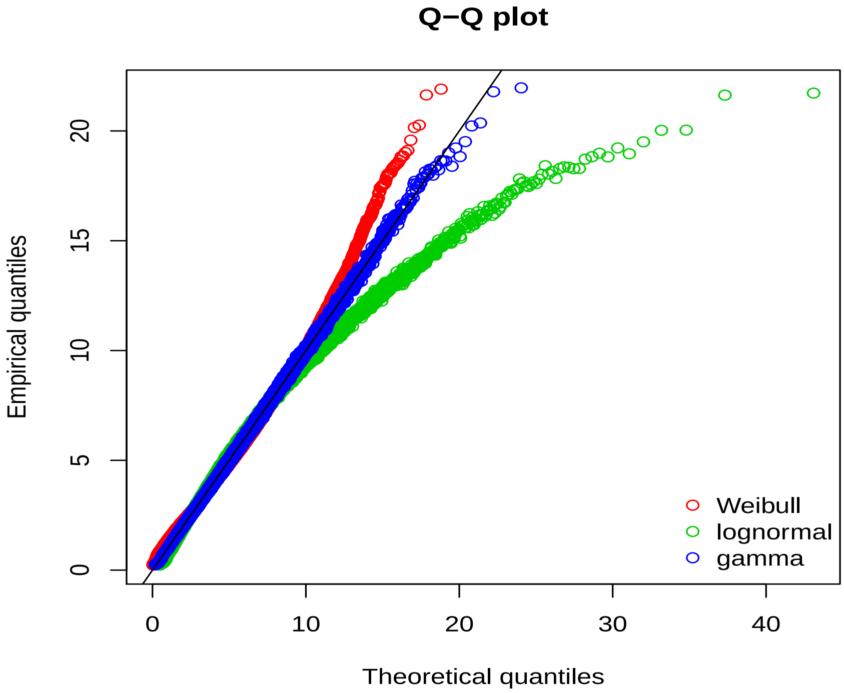

Figure 5.12: Q-Q plot for Gamma, Weibull and Lognormal for Kenya 2009 fertility data.

Again Gamma distribution fits the Kenya 2009 fertility data best.

CHAPTER 6

SUMMARY OF RESULTS, CONCLUSION AND RECOMENDATIONS

This chapter contains the summary of results obtained from modeling fertility rate of the data sets of Rwanda , Indonesia and Kenya both in the presence and also in the absence of interference. It also contains conclusions drawn from the summaries as well as the recommendations for future work.

6.1 Summary of Results

This study was set up with an objective of modeling the fertility rate data in the presence of interference. We determined the effect of interference in the fertility rates of Rwanda, Indonesia and Kenya by analyzing the fertility rate data sets of the mentioned countries before and after respective interferences which took place in the respective countries, and summarized our results as shown in Section 6.2 below. Using the life table approach, we modeled fertility rate by determining the Net fertility value, F_0 and linked it to population growth. We also fitted probability distributions to both the data sets which were interference free (Rwanda 1992, Indonesia 2002 and Kenya 2003) and also to those which had the interference effect in them (Rwanda 2000, Indonesia 2007 and Kenya 2009), and determined the effect of interference on the probability distributions.

6.2 Effect of interference on fertility rates of Rwanda, Indonesia and Kenya

We determined the effect of interference in the fertility rates of Rwanda, Indonesia and Kenya by analyzing the fertility rate data sets of Rwanda, Indonesia and Kenya before and after respective interferences which took place in the respective countries, and summarized our results in subsections 6.2.1 , 6.2.2 and 6.2.3.

6.2.1 Rwanda fertility rate before and after the 1994 Genocide Interference

Harff and Fein [15] refer to Genocide as any form of violent act committed with an aim of destroying in part or in whole, a national, racial or a religious group of people. It may be carried out through either killing members of the group, causing serious bodily or mental harm to members of the group, intentionally inflicting on the group, conditions of life calculated to bring about its physical destruction in whole or in part, imposing measures intended to prevent births within the group or even forcibly transferring children of the group to another group. The Rwanda Genocide, as was reported by Verwimp [49] in the year 2000 was a mass slaughter of Tutsi group by members of the Hutu majority, with an aim of eliminating them completely from existence. The violent act cleared 75 percent of the Tutsi group and claimed a total of 800,000 Rwandan lives. We carried out an analysis of the fertility rate of Rwanda in the years 1992, 2000 and 2005 DHS data sets [38, 39, 40], using R statistical software (see Appendix C.1.1) and displayed the results in Table 6.1 below.

Table 6.1: Fertility rate of Rwanda in the years 1992, 2000 and 2005

Year	Fertility rate
1992	5.885
2000	6.330
2005	5.876

From Table 6.1, the fertility rate of Rwanda in the year 1992 was 5.885. The rate then rose to 6.330 in the year 2000 and later on reduced to 5.876 in the year 2005. We observed from Table 6.1 that the fertility rate had increased in the Rwanda 2000 data set which had the Genocide interference effect.

6.2.2 Indonesia fertility rate before and after the 2004 Tsunami Interference

In the Macmillan English Dictionary [22], Tsunami is defined as a very large wave or series of waves caused when an earthquake moves a large quantity of water in the sea. The

World Health Organization [50] and Dudley [9] reported that a massive earthquake which measured 9.3 on the Richter scale struck the west of Northern Sumatra in 2004, triggered a powerful Tsunami which swept the coasts and neighboring countries such as Indonesia, Sri Lanka, India and Thailand, with Aceh province in Indonesia being the hardest hit and suffering the highest loss of life. The 2004 Indian Ocean Tsunami was exceptional in magnitude causing death toll of about 170,000 people and displacement of over 500,000 people in countries bordering the Indian ocean as was documented by Athukorala and Resosudarmo [2], Doocy [8] and [12]. We carried out an analysis of the fertility rate of Indonesia in the years 1997, 2002, 2007 and 2012 DHS data sets [31, 32, 33, 34], using R statistical software (see Appendix C.1.2) and displayed the results in Table 6.2 below.

Table 6.2: Fertility rate of Indonesia in the years 1997,2002, 2007 and 2012.

Year	Fertility rate
1997	2.995
2002	2.665
2007	3.930
2012	3.618

From Table 6.2, the fertility rate of Indonesia in the year 1997 was 2.995. This rate fell to 2.665 in the year 2002, then later rose up to 3.930 in 2007, and later on reduced to 3.618 in the year 2012. We observed from Table 6.2 that the fertility rate had increased in the Indonesia 2007 data set which had the Tsunami interference effect.

6.2.3 Kenya fertility rate before and after the 2008 Post election violence Interference

Samir [42] reported that in January 2008, Kenya underwent a post election violence following the 30th December, 2007 results of a hotly-contested presidential election. Opposition leader Raila Odinga and his supporters rejected the declared victory of incumbent president Mwai Kibaki, alleging it was the result of rigging. Protests went into widespread violence as decades of ethnic rivalry blew out of control. The violence claimed the lives of more than 1,200 people and about 600,000 people displaced into temporary camps. We carried out an analysis of the fertility rate of Kenya in the years 2003, 2009 and 2014 DHS data sets [35, 36, 37], using R statistical software (see Appendix C.1.3) and displayed the results in Table 6.3 below.

Table 6.3: Fertility rate of Kenya in the years 2003, 2009 and 2014.

Year	Fertility rate
2003	3.507
2009	5.277
2014	3.410

From Table 6.3, the fertility rate of Kenya in the year 2003 was 3.507. This rate rose up to 5.277 in the year 2009, and later on reduced to 3.410 in the year 2014. We observed from table 6.3 that the fertility rate had increased in the Kenya 2009 data set which had the Post election violence interference effect.

6.3 Determination of effect of interference on fertility rate

We fitted probability distribution functions to both the data sets which were interference free (Rwanda 1992, Indonesia 2002 and Kenya 2003) and also to those which had the interference effect in them (Rwanda 2000, Indonesia 2007 and Kenya 2009), and determined the effect of interference on the fertility rate data sets.

6.3.1 The Rwanda 1992 fertility findings

In Figure 4.1 we observed that the Rwanda 1992 fertility data was positively skewed . In Figure 4.2 we observed that the Kurtosis (2.44022) and skewness(0.2043342) of the data was consistent with either a Gamma, a Weibull or a Lognormal distribution. Among the three positively skewed distributions, Gamma distribution was observed to give the best fit (Figures 4.3 and 4.4). In addition, in Table 4.2 Gamma probability density function had fitted the Rwanda 1992 fertility data with the lowest AIC value hence was the best fitting density function with a shape parameter of 4.768 and a rate parameter of 0.810.

6.3.2 The Rwanda 2000 fertility findings

In Figure 5.1 we observed that the Rwanda 2000 fertility data was positively skewed . In Figure 5.2 we observed that the Kurtosis (2.536246) and skewness(0.1583683) of the data was consistent with either a Gamma, a Weibull or a Lognormal distribution. Among the three positively skewed distributions namely Gamma, Weibull and Lognormal that were fitted to the Rwanda 2000 fertility data in Figures 5.3 and 5.4 , Gamma distribution was observed to give the best fit. In addition, in Table 5.2 Gamma probability density function had fitted the Rwanda 2000 fertility data with the lowest AIC value hence was the best fitting density function with a shape parameter of 4.973 and a rate parameter of 0.823.

6.3.3 Gamma distribution fitted on Rwanda 1992 versus Gamma distribution fitted on Rwanda 2000 fertility rate data

The summary of the results from Table 4.2 and Table 5.2 were as follows;

Table 6.4: Summary of parameter estimates for Gamma fit on data sets of Rwanda 1992 and Rwanda 2000.

year	shape parameter	rate parameter
1992	4.768	0.810
2000	4.973	0.823

From Table 6.4, we observe an increase in both the shape (α) and the rate ($\frac{1}{\beta}$) parameters of the Gamma distribution.

The shape parameter α increased by 4.3 percent in the year 2000 compared to the year 1992.

The rate parameter $\frac{1}{\beta}$ increased by 1.6 percent in the year 2000 compared to the year 1992.

The Figure 6.1 shows a graph of Gamma distribution fitted to Rwanda 1992 and also to Rwanda 2000 fertility data sets on the same scale (see Appendix C.2.1)

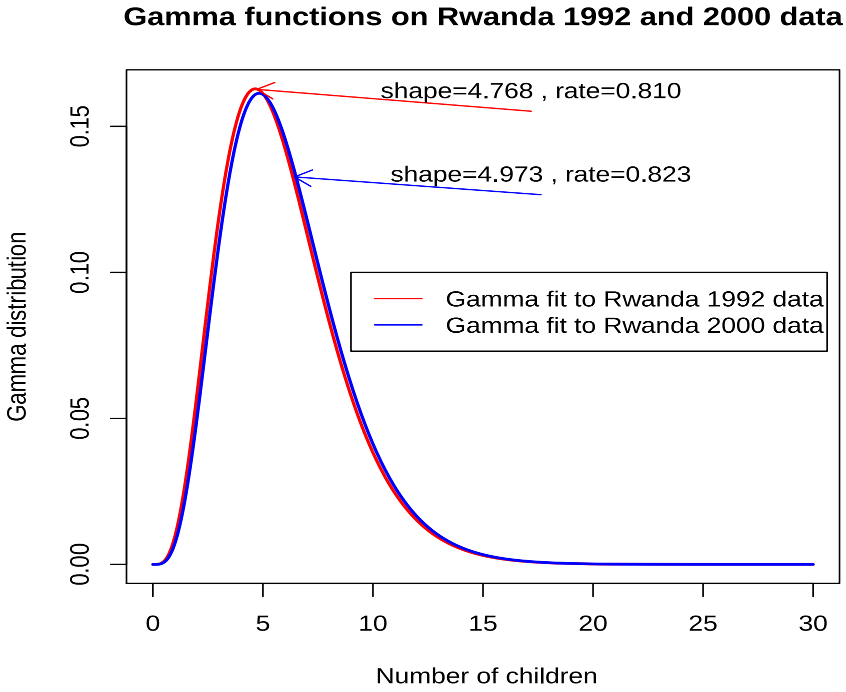

Figure 6.1: Gamma fit on Rwanda 1992 and on Rwanda 2000 data on the same scale

From Figure 6.1 above, the peakedness of the Gamma distribution fitted to Rwanda 2000 data had decreased and its range had also become broader as compared to the Gamma fitted to the Rwanda 1992 one.

6.3.4 The Indonesia 2002 fertility findings

In Figure 4.5 we observed that the Indonesia 2002 fertility data was positively skewed . In Figure 4.6 we observed that the Kurtosis (5.395964) and skewness(1.255609) of the data was consistent with either a Gamma, a Weibull or a Lognormal distribution. Among the three positively skewed distributions namely Gamma, Weibull and Lognormal that were fitted to the Indonesia 2002 fertility data in Figures 4.7 and 4.8 , Gamma distribution was observed to give the best fit. Also, in Table 4.4 Gamma probability density function had fitted the Indonesia 2002 fertility data with the lowest AIC value hence was the best fitting density function with a shape parameter of 2.022 and a rate parameter of 0.765.

6.3.5 The Indonesia 2007 fertility findings

In Figure 5.5 we observed that the Indonesia 2007 fertility data was positively skewed . In Figure 5.6 we observed that the Kurtosis (4.241901) and skewness(1.099586) of the data was consistent with either a Gamma, a Weibull or a Lognormal distribution. Gamma distribution was observed to give the best fit (Figures 5.7 and 5.8). In addition, in Table 5.4 Gamma probability density function had fitted the Indonesia 2007 fertility data with the lowest AIC value hence was the best fitting density function with a shape parameter of 3.091 and a rate parameter of 0.789.

6.3.6 Gamma distribution fitted on Indonesia 2002 versus Gamma distribution fitted on Indonesia 2007 fertility rate data.

The summary of the results from Table 4.4 and Table 5.4 were as follows;

Table 6.5: Summary of parameter estimates for Gamma fit on data sets of Indonesia 2002 and Indonesia 2007

year	shape parameter	rate parameter
2002	2.022	0.765
2007	3.091	0.789

From Table 6.5, we observe an increase in both the shape (α) and the rate ($\frac{1}{\beta}$) parameters of the Gamma distribution.

The shape parameter α increased by 52.9 percent in the year 2007 compared to the year 2002.

The rate parameter $\frac{1}{\beta}$ increased by 3.1 percent in the year 2007 compared to the year 2002.

Figure 6.2 below shows a graph of Gamma distribution fitted to Indonesia 2002 and also to Indonesia 2007 fertility data sets on the same scale (see Appendix C.2.2)

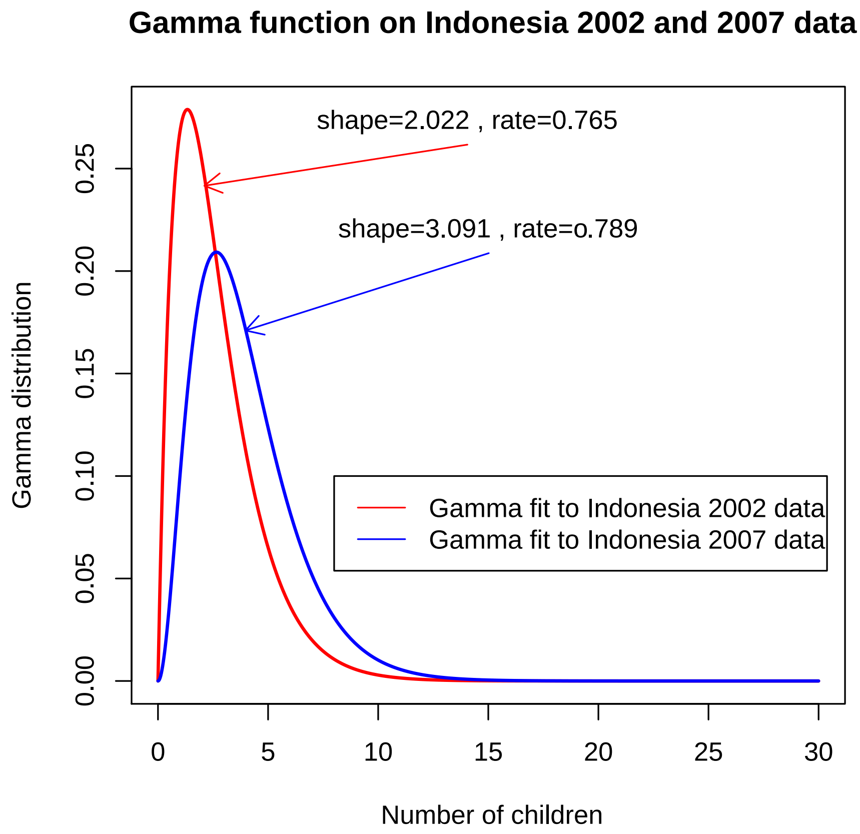

Figure 6.2: Gamma fit on Indonesia 2002 and on Indonesia 2007 data on the same scale

From Figure 6.2 above, the peakedness of the Gamma distribution fitted to Indonesia 2007 data had decreased and its range had become broader as compared to the Gamma fitted to the Indonesia 2002 one.

6.3.7 The Kenya 2003 fertility findings

In Figure 4.9 we observed that the Kenya 2003 fertility data was positively skewed . In Figure 4.10 we observed that the Kurtosis (4.051652) and skewness(1.12774) of the data was consistent with either a Gamma, a Weibull or a Lognormal distribution. Among the three positively skewed distributions namely Gamma, Weibull and Lognormal that were fitted to the Kenya 2003 fertility data in Figures 4.11 and 4.12 , Gamma distribution was observed to give the best fit. In addition, in Table 4.6 Gamma probability density function had fitted the Kenya 2003 fertility data with the lowest AIC value hence was the best fitting density function with a shape parameter of 1.989 and a rate parameter of 0.578.

6.3.8 The Kenya 2009 fertility findings

In Figure 5.9 we observed that the Kenya 2009 fertility data was positively skewed . In Figure 5.10 we observed that the Kurtosis (2.884296) and skewness(0.5378432) of the data was consistent with either a Gamma, a Weibull or a Lognormal distribution. Among the three positively skewed distributions namely Gamma, Weibull and Lognormal that were fitted to the Kenya 2009 fertility data in Figures 5.11 and 5.12 , Gamma distribution was observed to give the best fit. In addition, in Table 5.6 Gamma probability density function had fitted the Kenya 2009 fertility data with the lowest AIC value hence was the best fitting density function with a shape parameter of 3.771 and a rate parameter of 0.716.

6.3.9 Gamma distribution fitted on Kenya 2003 versus Gamma distribution fitted on Kenya 2009 fertility rate data.

The summary of the results from Table 4.6 and Table 5.6 were as follows;
Table 6.6: Summary of parameter estimates for Gamma
fit on data sets of Kenya 2003 and Kenya 2009

year	shape parameter	rate parameter
2003	1.989	0.578
2009	3.771	0.716

From Table 6.6, we observe an increase in both the shape and the rate parameters of the Gamma distribution.
The shape parameter α increased by 89.6 percent in the year 2009 compared to the year 2003.
The rate parameter $\frac{1}{\beta}$ increased by 23.7 percent in the year 2009 than in the year 2003.
The scale parameter β decreased by 19.1 percent in the year 2009 than in the year 2003.

From Table 6.6, we observe an increase in both the shape (α) and the rate ($\frac{1}{\beta}$) parameters of the Gamma distribution.

The shape parameter α increased by 89.6 percent in the year 2009 compared to the year 2003.

The rate parameter $\frac{1}{\beta}$ increased by 23.9 percent in the year 2009 compared to the year 2003.

The Figure 6.3 below shows a graph of Gamma distribution fitted to Kenya 2003 and also to Kenya 2009 fertility data sets on the same scale (see Appendix C.2.3)

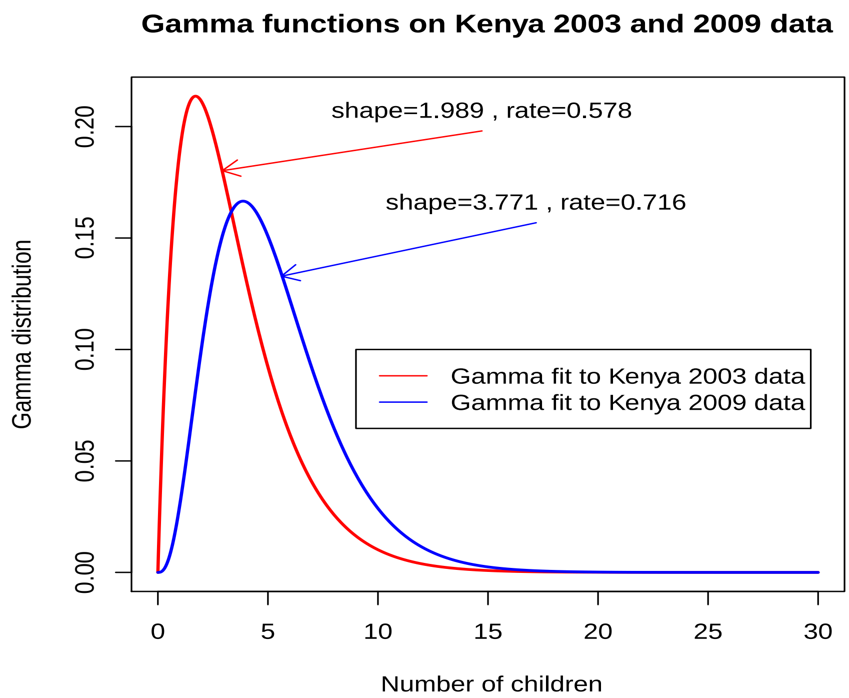

Figure 6.3: Gamma fit on Kenya 2003 and on Kenya 2009 data on the same scale

From Figure 6.3 above, the peakedness of the Gamma distribution fitted to Kenya 2009 data had decreased and its range had also become broader as compared to the Gamma fitted to the Kenya 2003 one.

6.4 Conclusions

In our study we set out to model the fertility rate in the presence of interference using five objectives. We managed to meet all our objectives as were clearly spelt out in Chapter one in Section 1.5.

we fitted the fertility data sets for Rwanda, Indonesia and Kenya were modeled before and after interference. The model parameters were estimated by the maximum likelihood estimation method. Using Akaike's Information Criteria, (AIC), it was established that amongst the distributions studied; Gamma, Weibull and Lognormal, Gamma gave the best fit for the fertility rate data, for all the countries studied, and interference simply shifts the Gamma distribution parameters (see Tables 6.4, 6.5 and 6.6). Also, in our analysis results in Tables 6.1, 6.2 and 6.3 fertility rates for all the countries studied had increased in the presence of interference effect. We concluded that presence of interference effect in a country increases its fertility rate.

Using the model life table approach we determined the Net Fertility Value, F_0 and concluded that F_0 is greater than 2 in case of interference effect, which is above replacement levels fertility of a population.

6.5 Recommendations

Strike of an interference in a country may create a period of baby boom (dramatic increase in fertility rates). Even though baby bust (rapid decline in fertility rates) normally follows after baby boom but as long as the baby boom period is there to stay, there should be adequate planning in the country affected to handle such a scenario. We therefore recommend that in case of a strike of interference, Governments should plan for it, by increasing the number of Health facilities to handle the increased need for antenatal and postnatal care programs. We also recommend that stake holders in a country should be quick to put its policies that govern population control in check soon after occurrence of an interference, so as to manage expected population increase due to baby boom.

6.6 Future work

We had an assumption in the model life table approach that population size and the sizes of age groups are all constant. In many situations, this assumption may not be correct causing bias in the results. The static nature of the life table may underestimate the the growth rate of a population as it fails to include the compensatory effects such as decrease in mortality and decrease in reproductive age. A study may be carried out which attempts to overcome the problem, of not including the compensation effects.

In this study we modeled fertility rate in the presence of interference. We considered the Tsunami, the Genocide and the Post election violence interferences. One could actually consider one type of interference , for example high magnitude Earthquake and develop a time series model for prediction purposes especially in a country where a series of high magnitude earthquakes have been experienced.

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Appendix A

R- MANUSCRIPT FOR MODELING INTERFERENCE FREE DATA SETS

A.1 Rwanda 1992 fertility data modeling

A.1.1 Plotting of Histogram for Rwanda 1992 fertility data

```
#Histogram plot
library(ggplot2)
qplot(rwanda1992$children,geom=‘histogram’,binwidth=1,xlab=‘Number
of children’,ylab=‘Frequency’,main=‘Histogram for Rwanda 1992
fertility data’,col=‘black’)
```

A.1.2 Skewness-Kurtosis plot for Rwanda 1992 fertility data

```
 # skewmess-kurtosis plot
library("fitdistrplus")
descdist(rwanda1992$children, boot = 1000)
```

A.1.3 Parameter estimates to Rwanda 1992 fertility data

```
library("fitdistrplus")
rwanda1992=read.table(‘C:/Users/EVA/Desktop/MY DATA/rwanda1992.txt’,
header=TRUE,strip.white=TRUE,na.strings=‘NA’,sep=‘\t’,dec=‘.’)
mean(rwanda1992$children)
var(rwanda1992$children)
m= mean(rwanda1992$children)
m
v=var(rwanda1992$children)
v
```

```
shape = m^2/v
shape
rate = m/v
rate
rwanda1992$children=rgamma(n=27602,shape=4.731041,rate=0.8039636)
#parameter estimates for weibull  for rwanda  1992 from sample
#rwanda1992=read.table('C:/Users/EVA/Desktop/MY DATA/rwanda1992.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
lx=log(rwanda1992$children)
m=mean(lx)
m
v=var(lx)
v
shape = m/sqrt(v)
shape
scale = exp(m + v/shape)
scale
#rwanda1992$children=rweibull(n=27602,shape=3.444908,scale=5.652191)
#parameter estimates for lognormal for rwanda  1992 from sample
rwanda1992=read.table('C:/Users/EVA/Desktop/MY DATA/rwanda1992.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
lx=log(rwanda1992$children)
sd0 = sqrt((n - 1)/n) * sd(lx)
sd0
mx=mean(lx)
mx
estimate =c(mx, sd0)
estimate
```

A.1.4 Fitting Gamma , Weibull and Lognormal distributions to Rwanda 1992 fertility data

```
rwanda1992$children=rgamma(n=27602,shape=4.731041,rate=0.8039636)
library(‘‘fitdistrplus")
fg <- fitdist(rwanda1992$children, ‘‘gamma")
summary(fg)
fw <- fitdist(rwanda1992$children, ‘‘weibull")
summary(fw)
fln <- fitdist(rwanda1992$children, ‘‘lnorm")
summary(fln)
```

A.1.5 Quality and goodness of fit to Rwanda 1992 fertility data

```
#Evaluation of quality of fit for rwanda 1992 data
rwanda1992$children=rgamma(n=27602,shape=4.731041,rate=0.8039636)
library(‘‘fitdistrplus")
#par(mfrow=c(2,1))
plot.legend <- c(‘‘Weibull", ‘‘lognormal", ‘‘gamma")
denscomp(list(fw,fln,fg),legendtext=plot.legend,xlab=‘Number of children’,
ylab=‘Density’,main=c(‘Histogram and some probability densities’,‘
on Rwanda 1992 fertility data’))
qqcomp(list(fw,fln,fg),legendtext=plot.legend,main=c(‘Q-Q plot for some
 probability densities’,‘ on Rwanda 1992 fertility data’))
```

A.2 Indonesia 2002 fertility data modeling

A.2.1 Plotting of Histogram for Indonesia 2002 fertility data

```
indonesia2002=read.table(‘C:/Users/EVA/Desktop/MY DATA/indonesia2002.txt’
,header=TRUE,strip.white=TRUE,na.strings=‘NA’,sep=‘\t’,dec=‘.’)
#Histogram plot
library(ggplot2)
```

```
qplot(indonesia2002$children,geom='histogram',binwidth=1,xlab='Number of
 children',ylab='Frequency',main='Histogram for Indonesia 2002 fertility
 data',col='black')
```

A.2.2 Skewness-Kurtosis plot for Indonesia 2002 fertility data

```
 # skewmess-kurtosis plot
library("fitdistrplus")
descdist(indonesia2002$children, boot = 1000)
```

A.2.3 Parameter estimates to Indonesia 2002 fertility data

```
#Parameter estimates for gamma,Weibull and Lognormal for Indonesia 2002
#parameter estimates for gamma for indonesia 2002 from sample
indonesia2002=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2002.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
mean(indonesia2002$children)
var(indonesia2002$children)
m= mean(indonesia2002$children)
m
v=var(indonesia2002$children)
v
shape = m^2/v
shape
rate = m/v
rate
indonesia2002$children=rgamma(n=7684,shape=1.974798,rate=0.7410074)
#parameter estimates for weibull for indonesia 2002 from sample
#indonesia2002=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2002.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
indonesia2002$children=rgamma(n=7684,shape=1.974798,rate=0.7410074)
lx=log(indonesia2002$children)
```

```
m=mean(lx)
m
v=var(lx)
v
shape = m/sqrt(v)
shape
scale = exp(m + v/shape)
scale
#parameter estimates for lognormal for indonesia 2002 from sample
#indonesia2002=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2002.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
indonesia2002$children=rgamma(n=7684,shape=1.974798,rate=0.7410074)
lx=log(indonesia2002$children)
sd0 = sqrt((n - 1)/n) * sd(lx)
sd0
mx=mean(lx)
mx
estimate =c(mx, sd0)
estimate
```

A.2.4 Fitting Gamma , Weibull and Lognormal distributions to Indonesia 2002 fertility data

```
#Fitting Gamma , Weibull and Lognormal distributions to Indonesia 2002 data
indonesia2002$children=rgamma(n=7684,shape=1.974798,rate=0.7410074)
library("fitdistrplus")
fg <- fitdist(indonesia2002$children, "gamma")
summary(fg)
fw <- fitdist(indonesia2002$children, "weibull")
summary(fw)
fln <- fitdist(indonesia2002$children, "lnorm")
```

```
summary(fln)
```

A.2.5 Quality and goodness of fit to Indonesia 2002 fertility data

```
#Evaluation of quality of fit for indonesia 2002
indonesia2002$children=rgamma(n=7684,shape=1.974798,rate=0.7410074)
library("fitdistrplus")
#par(mfrow=c(2,1))
plot.legend <- c("Weibull", "lognormal", "gamma")
denscomp(list(fw,fln,fg),legendtext=plot.legend,xlab='Number of
 children',ylab='Density',main=c('Histogram and
 some probability densities',' on Indonesia 2002 fertility data'))
qqcomp(list(fw,fln,fg),legendtext=plot.legend,main=c('Q-Q plot
for some probability densities',' on Indonesia 2002 fertility data'))
```

A.3 Kenya 2003 fertility data modeling

A.3.1 Plotting of Histogram for Kenya 2003 fertility data

```
kenya2003=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2003.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
#Histogram plot
library(ggplot2)
qplot(kenya2003$children,geom='histogram',binwidth=1,xlab='Number of
children',ylab='Frequency',main='Histogram for Kenya 2003 fertility
 data',col='black')
```

A.3.2 Skewness-Kurtosis plot for Kenya 2003 fertility data

```
 # skewmess-kurtosis plot
library("fitdistrplus")
descdist(kenya2003$children, boot = 1000)
```

A.3.3 Parameter estimates to Kenya 2003 fertility data

```
#Parameter estimates for gamma, Weibull and Lognormal for Kenya 2003 data
#parameter estimates for gamma for kenya 2003 from sample
kenya2003=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2003.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
mean(kenya2003$children)
var(kenya2003$children)
m= mean(kenya2003$children)
m
v=var(kenya2003$children)
v
shape = m^2/v
shape
rate = m/v
rate
kenya2003$children=rgamma(n=1430,shape=1.908814,rate=0.5442879)
#parameter estimates for weibull for kenya 2003 from sample
#kenya2003=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2003.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
kenya2003$children=rgamma(n=1430,shape=1.908814,rate=0.5442879)
lx=log(kenya2003$children)
m=mean(lx)
m
v=var(lx)
v
shape = m/sqrt(v)
shape
scale = exp(m + v/shape)
scale
#parameter estimates for lognormal for kenya 2003 from sample
```

```
#kenya2003=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2003.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
kenya2003$children=rgamma(n=1430,shape=1.908814,rate=0.5442879)
lx=log(kenya2003$children)
sd0 = sqrt((n - 1)/n) * sd(lx)
sd0
mx=mean(lx)
mx
estimate =c(mx, sd0)
estimate
```

A.3.4 Fitting Gamma , Weibull and Lognormal distributions to Kenya 2003 fertility data

```
#Fitting Gamma , Weibull and Lognormal distributions to Kenya 2003 data
kenya2003$children=rgamma(n=1430,shape=1.908814,rate=0.5442879)
library("fitdistrplus")
fg <- fitdist(kenya2003$children, "gamma")
summary(fg)
fw <- fitdist(kenya2003$children, "weibull")
summary(fw)
fln <- fitdist(kenya2003$children, "lnorm")
summary(fln)
```

A.3.5 Quality and goodness of fit to Kenya 2003 fertility data

```
#Evaluation of quality of fit for Kenya 2003
kenya2003$children=rgamma(n=1430,shape=1.908814,rate=0.5442879)
library("fitdistrplus")
plot.legend <- c("Weibull", "lognormal", "gamma")
denscomp(list(fw,fln,fg),legendtext=plot.legend,xlab='Number of children',
ylab='Density',main=c('Histogram and some probability densities','
```

```
on Kenya 2003 fertility data'))
qqcomp(list(fw,fln,fg),legendtext=plot.legend,main=c('Q-Q plot for some
probability densities',' on Kenya 2003 fertility data'))
```

Appendix B

R- MANUSCRIPT FOR MODELING DATA CONTAINING INTERFERENCE

B.1 Rwanda 2000 fertility data modeling

B.1.1 Plotting of Histogram for Rwanda 2000 fertility data

```
rwanda2000=read.table(‘C:/Users/EVA/Desktop/MY DATA/rwanda2000.txt’,
header=TRUE,strip.white=TRUE,na.strings=‘NA’,sep=‘\t’,dec=‘.’)
# Histogram plot
library(ggplot2)
qplot(rwanda2000$children,geom=‘histogram’,binwidth=1,xlab=‘Number of
children’,ylab=’Frequency’,main=’Histogram for Rwanda 2000 fertility
data’,col=‘black’)
```

B.1.2 Skewness-Kurtosis plot for Rwanda 2000 fertility data

```
 # skewmess-kurtosis plot
library("fitdistrplus")
descdist(rwanda2000$children, boot = 1000)
```

B.1.3 Parameter estimates to Rwanda 2000 fertility data

```
#Parameter estimates for gamma, Weibull and Lognormal for Rwanda 2000 data
#Parameter estimates for gamma for rwanda 2000 data
library("fitdistrplus")
rwanda2000=read.table(‘C:/Users/EVA/Desktop/MY DATA/rwanda2000.txt’,
header=TRUE,strip.white=TRUE,na.strings=‘NA’,sep=‘\t’,dec=‘.’)
mean(rwanda2000$children)
var(rwanda2000$children)
```

```
m= mean(rwanda2000$children)
m
v=var(rwanda2000$children)
v
shape = m^2/v
shape
rate = m/v
rate
rwanda2000$children=rgamma(n=19440,shape=4.946252,rate=0.7813425)
#descdist(rwanda2000$children, boot = 1000)
#parameter estimates for weibull from rwanda 2000 from sample
#rwanda2000=read.table('C:/Users/EVA/Desktop/MY DATA/rwanda2000.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
lx=log(rwanda2000$children)
m=mean(lx)
m
v=var(lx)
v
shape = m/sqrt(v)
shape
scale = exp(m + v/shape)
scale
#rwanda2000$children=rweibull(n=19440,shape=4.946252,rate=0.7813425)
#parameter estimates for lognormal from rwanda 2000 from sample
#rwanda2000=read.table('C:/Users/EVA/Desktop/MY DATA/rwanda2000.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
lx=log(rwanda2000$children)
sd0 = sqrt((n - 1)/n) * sd(lx)
sd0
mx=mean(lx)
```

```
mx
estimate =c(mx, sd0)
estimate
```

B.1.4 Fitting Gamma , Weibull and Lognormal distributions to Rwanda 2000 fertility data

```
#Fitting Gamma , Weibull and Lognormal distributions to Rwanda 2000 data
rwanda2000$children=rgamma(n=19440,shape=4.946252,rate=0.7813425)
library("fitdistrplus")
fg <- fitdist(rwanda2000$children, "gamma")
summary(fg)
fw <- fitdist(rwanda2000$children, "weibull")
summary(fw)
fln <- fitdist(rwanda2000$children, "lnorm")
summary(fln)
```

B.1.5 Quality and goodness of fit to Rwanda 2000 fertility data

```
#Evaluation of quality of fit to Rwanda 2000
rwanda2000$children=rgamma(n=19440,shape=4.946252,rate=0.7813425)
library("fitdistrplus")
plot.legend <- c("Weibull", "lognormal", "gamma")
denscomp(list(fw,fln,fg),legendtext=plot.legend,xlab=‘Number of children’,
ylab=‘Density’,main=c(‘Histogram and some probability densities’,‘
on Rwanda 2000 fertility data’))
qqcomp(list(fw,fln,fg),legendtext=plot.legend,main=c(‘Q-Q plot for some
probability densities’,‘ on Rwanda 2000 fertility data’))
```

B.2 Indonesia 2007 fertility data modeling

B.2.1 Plotting of Histogram for Indonesia 2007 fertility data

```
indonesia2007=read.table(‘C:/Users/EVA/Desktop/MY DATA/indonesia2007.txt’
```

```
,header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
# Histogram plot
library(ggplot2)
qplot(indonesia2007$children,geom='histogram',binwidth=1,xlab='Number
 of children',ylab='Frequency',main='Histogram for Indonesia 2007
  fertility data',col='black')
```

B.2.2 Skewness-Kurtosis plot for Indonesia 2007 fertility data

```
 # skewmess-kurtosis plot
library("fitdistrplus")
descdist(indonesia2007$children, boot = 1000)
```

B.2.3 Parameter estimates to Indonesia 2007 fertility data

```
#Parameter estimates for gamma, Weibull and Lognormal for Indonesia 2007
#Parameter estimates for gamma for indonesia 2007 data
library("fitdistrplus")
#parameter estimates for indonesia 2007 from sample
indonesia2007=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2007.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
mean(indonesia2007$children)
var(indonesia2007$children)
m= mean(indonesia2007$children)
m
v=var(indonesia2007$children)
v
shape = m^2/v
shape
rate = m/v
rate
indonesia2007$children=rgamma(n=84726,shape=3.09893,rate=0.7885535)
```

```
#parameter estimates for weibull for indonesia 2007 from sample
#indonesia2007=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2007.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
indonesia2007$children=rgamma(n=84726,shape=3.09893,rate=0.7885535)
lx=log(indonesia2007$children)
m=mean(lx)
m
v=var(lx)
v
shape = m/sqrt(v)
shape
scale = exp(m + v/shape)
scale
#parameter estimates for lognormal for indonesia 2007 from sample
#indonesia2007=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2007.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
indonesia2007$children=rgamma(n=84726,shape=3.09893,rate=0.7885535)
lx=log(indonesia2007$children)
sd0 = sqrt((n - 1)/n) * sd(lx)
sd0
mx=mean(lx)
mx
estimate =c(mx, sd0)
estimate
```

B.2.4 Fitting Gamma , Weibull and Lognormal distributions to Indonesia 2007 fertility data

```
#Fitting Gamma , Weibull and Lognormal distributions to Indonesia 2002
indonesia2007$children=rgamma(n=84726,shape=3.09893,rate=0.7885535)
library("fitdistrplus")
```

```
fg <- fitdist(indonesia2007$children, "gamma")
summary(fg)
fw <- fitdist(indonesia2007$children, "weibull")
summary(fw)
fln <- fitdist(indonesia2007$children, "lnorm")
summary(fln)
```

B.2.5 Quality and goodness of fit to Indonesia 2007 fertility data

```
#Evaluation of quality of fit to Indonesia 2007
indonesia2007$children=rgamma(n=84726,shape=3.09893,rate=0.7885535)
library("fitdistrplus")
plot.legend <- c("Weibull", "lognormal", "gamma")
denscomp(list(fw,fln,fg),legendtext=plot.legend,xlab='Number of children',
ylab='Density',main=c('Histogram and some probability densities',' on
Indonesia 2007 fertility data'))
qqcomp(list(fw,fln,fg),legendtext=plot.legend,main=c('Q-Q plot for some
 probability densities',' on Indonesia 2007 fertility data'))
```

B.3 Kenya 2009 fertility data modeling

B.3.1 Plotting of Histogram for Kenya 2009 fertility data

```
kenya2009=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2009.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
# Histogram plot
library(ggplot2)
qplot(kenya2009$children,geom='histogram',binwidth=1,xlab='Number of
 children',ylab='Frequency',main='Histogram for Kenya 2009
  fertility data',col='black')
```

B.3.2 Skewness-Kurtosis plot for Kenya 2009 fertility data

```
 # skewmess-kurtosis plot
```

```
library("fitdistrplus")
descdist(kenya2009$children, boot = 1000)
```

B.3.3 Parameter estimates to Kenya 2009 fertility data

```
#Parameter estimates for gamma, Weibull and Lognormal for Kenya 2009 data
#Parameter estimates for gamma for Kenya 2009 data
library("fitdistrplus")
#parameter estimates for gamma for kenya 2009 from sample
var(kenya2009$children)
m= mean(kenya2009$children)
m
v=var(kenya2009$children)
v
shape = m^2/v
shape
rate = m/v
rate
kenya2009$children=rgamma(n=22534,shape=3.845601,rate=0.7286859)
#parameter estimates for weibull for kenya 2009 from sample
#kenya2009=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2009.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
kenya2009$children=rgamma(n=22534,shape=3.845601,rate=0.7286859)
lx=log(kenya2009$children)
m=mean(lx)
m
v=var(lx)
v
shape = m/sqrt(v)
shape
scale = exp(m + v/shape)
```

```
scale
#parameter estimates for lognormal for kenya 2009 from sample
#kenya2009=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2009.txt'
,header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
kenya2009$children=rgamma(n=22534,shape=3.845601,rate=0.7286859)
lx=log(kenya2009$children)
sd0 = sqrt((n - 1)/n) * sd(lx)
sd0
mx=mean(lx)
mx
estimate =c(mx, sd0)
estimate
```

B.3.4 Fitting Gamma , Weibull and Lognormal distributions to Kenya2009 fertility data

```
#Fitting Gamma , Weibull and Lognormal distributions to Kenya 2009 data
kenya2009$children=rgamma(n=22534,shape=3.845601,rate=0.7286859)
library("fitdistrplus")
fg <- fitdist(kenya2009$children, "gamma")
summary(fg)
fw <- fitdist(kenya2009$children, "weibull")
summary(fw)
fln <- fitdist(kenya2009$children, "lnorm")
summary(fln)
```

B.3.5 Quality and goodness of fit to Kenya 2009 fertility data

```
#Evaluation of quality of fit to Kenya 2009
kenya2009$children=rgamma(n=22534,shape=3.845601,rate=0.7286859)
library("fitdistrplus")
denscomp(list(fw,fln,fg),legendtext=plot.legend,xlab='Number of children',
```

```
ylab='Density',main=c('Histogram and some probability densities','
on Kenya 2009 fertility data'))
qqcomp(list(fw,fln,fg),legendtext=plot.legend,main=c('Q-Q plot for some
 probability densities',' on Kenya 2009 fertility data'))
```

Appendix C

R- MANUSCRIPT FOR SUMMARY OF RESULTS, CONCLUSIONS AND RECOMENDATIONS

C.1 Analysis of the Rwanda, Indonesia and Kenya fertility data

C.1.1 Analysis of the 1992, 2000 and 2005 Rwanda fertility data

```
#analysis of the mean number of children born in Rwanda ,
rwanda1992=read.table('C:/Users/EVA/Desktop/MY DATA/rwanda1992.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(rwanda1992$children)
rwanda2000=read.table('C:/Users/EVA/Desktop/MY DATA/rwanda2000.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(rwanda2000$children)
rwanda2005=read.table('C:/Users/EVA/Desktop/MY DATA/rwanda2005.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(rwanda2005$children)
```

C.1.2 Analysis of the 1997, 2002, 2007 and 2012 Indonesia fertility data

```
#analysis of the mean number of children born in Indonesia
indonesia1997=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia1997.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(indonesia1997$children)
indonesia2002=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2002.
txt',header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(indonesia2002$children)
indonesia2007=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2007.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(indonesia2007$children)
```

```
indonesia2012=read.table('C:/Users/EVA/Desktop/MY DATA/indonesia2012.
txt',header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(indonesia2012$children)
```

C.1.3 Analysis of the 2003, 2009 and 2014 Kenya fertility data

```
#analysis of the mean number of children born in Kenya ,
kenya2003=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2003.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(kenya2003$children)
kenya2009=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2009.
txt',header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(kenya2009$children)
kenya2014=read.table('C:/Users/EVA/Desktop/MY DATA/kenya2014.txt',
header=TRUE,strip.white=TRUE,na.strings='NA',sep='\t',dec='.')
summary(kenya2014$children)
```

C.2 Determination of effect interference on the probability distributions in presence of interference

C.2.1 Gamma fitted to Rwanda 1992 and Rwanda 2000 fertility data sets on same scale

```
rwanda1992=rgamma(n=1000,shape=4.731041,rate=0.8039636)
rwanda2000=rgamma(n=1000,shape=4.946252,rate=0.8113425)
eta=seq(0,30,length=1000)
rwanda1992=seq(0, 30, length=1000)
rwanda2000=seq(0, 30, length=1000)
simmapDefaultGamma1 <- dgamma(rwanda1992, shape=4.731041, rate=0.8039636)
simmapDefaultGamma2 <- dgamma(rwanda2000, shape=4.946252, rate=0.81134)
#Make probability density function for SIMMAP default gamma distribution
plot(c(0,30),range(simmapDefaultGamma1 ,simmapDefaultGamma2 ),xlab='
Number of children',ylab='Gamma distribution',main='Gamma functions on
```

```
Rwanda 1992 and 2000 data',type='n',axes=FALSE)
axis(2)
box()
lines(eta,simmapDefaultGamma1,lwd=2,col='red')
lines(eta,simmapDefaultGamma2,lwd=2,col='blue')
coords = locator(2)
arrows(coords$x[1], coords$y[1], coords$x[2], coords$y[2], code=1,
length=0.125,col='red')
text(coords$x[2], coords$y[2], pos=3, "shape=4.731 , rate=0.804")
coords = locator(2)
arrows(coords$x[1], coords$y[1], coords$x[2], coords$y[2], code=1,
length=0.125,col='blue')
text(coords$x[2], coords$y[2], pos=3, "shape=4.946 , rate=0.811")
legend(9,0.1,lwd=1,col=c('red','blue'),legend=c('Gamma fit to
Rwanda 1992 data','Gamma fit to Rwanda 2000 data'))
```

C.2.2 Gamma fitted to Indonesia 2002 and Indonesia 2007 fertility data sets on same scale

```
indonesia2002=rgamma(n=1000,shape=1.974798,rate=0.7410074)
indonesia2007=rgamma(n=1000,shape=3.09893,rate=0.7885535)
eta=seq(0,30,length=1000)
indonesia2002=seq(0, 30, length=1000)
indonesia2007=seq(0, 30, length=1000)
simmapDefaultGamma1 <- dgamma(indonesia2002, shape=1.974798,
rate=0.7410074)
simmapDefaultGamma2 <- dgamma(indonesia2007, shape=3.09893,rate=0.7885535)
#Make probability density function for SIMMAP default gamma distribution
plot(c(0,30),range(simmapDefaultGamma1 ,simmapDefaultGamma2 ),xlab='
Number of children',ylab='Gamma distribution',main=' Gamma function on
 Indonesia 2002 and 2007 data',type='n',axes=FALSE)
```

```
axis(2)
box()
lines(eta,simmapDefaultGamma1,lwd=2,col='red')
lines(eta,simmapDefaultGamma2,lwd=2,col='blue')
coords = locator(2)
arrows(coords$x[1], coords$y[1], coords$x[2], coords$y[2], code=1,
length=0.125,col='red')
text(coords$x[2], coords$y[2], pos=3, "shape=1.975 , rate=0.741")
coords = locator(2)
arrows(coords$x[1], coords$y[1], coords$x[2], coords$y[2], code=1,
length=0.125,col='blue')
text(coords$x[2], coords$y[2], pos=3, "shape=3.099 , rate=o.789")
legend(8,0.1,lwd=1,col=c('red','blue'),legend=c('Gamma fit to
 Indonesia 2002 data','Gamma fit to Indonesia 2007 data'))
```

C.2.3 Gamma fitted to Kenya 2003 and Kenya 2009 fertility data sets on same scale

```
kenya2003=rgamma(n=1000,shape=1.908814,rate=0.5442879)
kenya2009=rgamma(n=1000,shape=3.845601,rate=0.7286859)
eta=seq(0,30,length=1000)
kenya2003=seq(0, 30, length=1000)
kenya2009=seq(0, 30, length=1000)
simmapDefaultGamma1 <- dgamma(kenya2003, shape=1.908814, rate=0.5442879)
simmapDefaultGamma2 <- dgamma(kenya2009, shape=3.845601, rate=0.7286859)
#Make probability density function for SIMMAP default gamma distribution
plot(c(0,30),range(simmapDefaultGamma1 ,simmapDefaultGamma2 ),xlab='
Number of children',ylab='Gamma distribution',main='Gamma functions on
 Kenya 2003 and 2009 data',type='n',axes=FALSE)
axis(2)
box()
```

```
lines(eta,simmapDefaultGamma1,lwd=2,col='red')
lines(eta,simmapDefaultGamma2,lwd=2,col='blue')
coords = locator(2)
arrows(coords$x[1], coords$y[1], coords$x[2], coords$y[2], code=1,
length=0.125,col='red')
text(coords$x[2], coords$y[2], pos=3, "shape=1.909 , rate=0.544")
coords = locator(2)
arrows(coords$x[1], coords$y[1], coords$x[2], coords$y[2], code=1,
length=0.125,col='blue')
text(coords$x[2], coords$y[2], pos=3, "shape=3.846 , rate=0.729")
legend(9,0.1,lwd=1,col=c('red','blue'),legend=c('Gamma fit to Kenya
 2003 data','Gamma fit to Kenya 2009 data'))
```